

# Thornhill Primary School Policy for Calculation

The policy was developed by the Curriculum Leader for Maths and reviewed by staff and SLT and agreed in September 2020.

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new National Curriculum. Through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. Children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

### Aims:

The aim of the calculation policy is to ensure consistency throughout the school and to make teachers aware of the continuity and progression in skill development across the year groups. It enables staff, and other stakeholders where relevant, to see how the concepts, facts and calculation strategies are applied and taught in each year group.

This is to enable children to calculate successfully and be able to:

have a confident and competent understanding of numbers and the number system

have instant recall of a set of basic number facts (fluency)

use a range of mental calculation strategies effectively, efficiently and appropriately

use and apply all of the above to solve problems and reason mathematically.

The above aims are linked to the Mathematics policy aims which are:

To develop knowledge and understanding of mathematical ideas, processes and skills and relate these to everyday experiences.

To acquire and refine mathematical and practical skills necessary to investigate ideas and questions. To develop a progressive understanding of mathematical concepts, skills and attitudes appropriate to all children's abilities.

To promote mathematics across the curriculum where appropriate and where it enhances mathematical application.

### Children deserve:

To be set appropriate learning challenges.

To be taught well and be given the opportunity to learn in ways that maximise the chances of success. To have support to enable them to tackle the specific barriers to progress they face.

### **Objectives:**

We will achieve these aims by the following objectives:

To provide pupils with the opportunity to develop their mathematical skills to the best of their ability. To develop their skills and potential within a broad curriculum to include practical tasks, problem solving and real life situations.

To develop the ability to select and use appropriate mathematical equipment and materials.

To create an enthusiastic and positive attitude towards the learning and application of the subject.

### Organisation

### **Teaching & Learning**

### The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

### To add and subtract successfully, children should be able to:

- recall all addition pairs to 9 + 9 and number bonds to 10
- · recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

### To multiply and divide successfully, children should be able to:

- · add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to  $144 \div 12 = 12$
- · use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- · understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that  $15 = 3 \times 5$ , or that  $40 = 10 \times 4$ ) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- · partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- · understand the effects of scaling by whole numbers and decimal numbers or fractions
- · understand correspondence where n objects are related to m objects
- · investigate and learn rules for divisibility

See the following pages for pedagogy and specific examples of progression.

# Appendix



# NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

count, read and write numbers from 1 to 20 in numerals and words

count, read and write numbers to 100 in numerals

count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number

count in multiples of twos, fives and tens

given a number, identify one more and one less

compare and order numbers to at least 20

identify and represent numbers using objects and pictorial representations, including the number line, and use the language of: equal to, more than, less than (few er), most, least

# ADDITION

### Conceptual understanding and procedural fluency To add successfully, pupils need to: understand addition as combining two or more groups of objects understand addition as counting on represent and use number bonds within 20 add one-digit and two-digit numbers to 20, including zero realise the effect of adding zero recall doubles of all numbers to 10 understand that addition can be done in any order read, write and interpret mathematical statements involving addition (+) and equals (=) signs Reason mathematically and solve problems Pupils need to use and apply their understanding of, and fluency in, addition to: solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 16 = - + 7solve one-step problems that involve addition in familiar contexts, e.g. money **Mental strategies** Use of models and images: concrete objects/pictorial representations $\bigcirc$ $\bigcirc \bigcirc$ number tracks and number lines 1 2 3 0 Λ 5 1-100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	<b>2</b> 3	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use of resources such as Numicon

Identify near doubles, using doubles already known (e.g. 6 + 5)

Recognise and use patterns of similar calculations (e.g. 10 + 0 = 10, 9 + 1 = 10, 8 + 2 = 10...) Understand and use the inverse relationship between addition and subtraction

### SUBTRACTION



1	1	2	3	4	5	6	7	8	9	
11	1	12	13	14	15	16	17	18	19	
21	1	22	23	24	25	26	27	28	29	
31	1	32	33	34	35	36	37	38	39	
41	1	42	43	44	45	<b>4</b> 6	47	48	49	
51	1	52	53	54	55	56	57	58	59	
61	1	62	63	64	65	66	67	68	69	
71	'1	72	73	74	75	76	77	78	79	
81	1	82	83	84	85	86	87	88	8 <del>9</del>	
91	1	92	93	94	95	96	97	98	99	
ogni	ise	e an	id us	se p	atter	ns c	ofsin	milar	cal	culat
(e.g.	1(	0 - 0	0 = 1	10, 1	10 –	1 =	9, 1	0 – 2	2 = 8	3)
dersta	tan	id a	nd เ	ise t	he i	nver	se r	elati	onsh	nip b

### **MULTIPLICATION**

Conceptual understanding and procedural fluency
To multiply successfully, pupils need to: understand multiplication through grouping small quantities understand the link betw een multiplication and doubling
Reason mathematically and solve problems
Pupils need to use and apply their understanding of, and fluency in, multiplication to: solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
solve one-step problems that involve multiplication in familiar contexts
Mental strategies
Use of models and images:
- concrete objects/pictorial representations
$- \frac{\text{arrays}}{\star \star \star \star \star} 4 \text{ lots of 5 is 20}$
5 lots of 4 is 20
Make connections between arrays, number patterns and counting in steps of a constant size

### DIVISION

### Conceptual understanding and procedural fluency

To divide successfully, pupils need to: understand division through sharing small quantities understand the link between division and halving

### Reason mathematically and solve problems Pupils need to use and apply their understanding of, and fluency in, division to: solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher solve one-step problems that involve division in familiar contexts **Mental strategies** Use of models and images: concrete objects/pictorial representations array \* \* \* 4 lots of 5 is 20 \* \* \* \* \* 5 lots of 4 is 20 Make connections between arrays, number patterns and counting in steps of a constant size



# NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read and write numbers to at least 100 in numerals and in words count in steps of 2, 3, and 5 from 0, and in tens from any number, forwards and backwards recognise the place value of each digit in a two-digit number (tens, ones) identify, represent and estimate numbers using different representations, including the number line compare and order numbers from 0 up to 100; use <, > and = signs

# ADDITION

### Conceptual understanding and procedural fluency

To add successfully, pupils need to:

recall and use addition facts to 20 fluently, and derive and use related facts up to 100, including adding two multiples of 10, e.g. 30 + 50

add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- twotwo-digit numbers
- three one-digit numbers

show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

record addition in columns to support place value and prepare for the formal written method with larger numbers

#### Reason mathematically and solve problems



#### Expanded written method

38+25

38 + <u>25</u> 13 <u>50</u> 63

Record addition calculations in columns to support place value and prepare for the formal written method of columnar addition with larger numbers.

The first stage in the written method shows separately the addition of the ones to the ones and the tens to the tens. To find the partial sums either the ones or the tens can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method.

The addition of the tens in the calculation 38 + 25 is described in the words 'thirty add tw enty equals fifty', stressing the link to the related fact 'three add tw o equals five'.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

# SUBTRACTION

### Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100, including subtracting two multiples of 10, e.g. 80 - 30

subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- twotwo-digit numbers

show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

record subtraction in columns to support place value and prepare for the formal written method with larger numbers

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to: solve problems with subtraction:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods

#### **Mental strategies**







# DIVISION

#### Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

recognise division as grouping or sharing

recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

understand the link between division and fractions, and find fractions of a length, shape, set of objects or quantity calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs







# NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read and write numbers up to 1000 in numerals and in words count from 0 in multiples of 1, 2, 3, 4, 5, 8, 10, 50 and 100, forwards and backwards recognise the place value of each digit in a three-digit number (hundreds, tens, ones) identify, represent and estimate numbers using different representations find 10 or 100 more or less than a given number compare and order numbers up to 1000

# ADDITION

#### Conceptual understanding and procedural fluency

#### To add successfully, pupils need to:

continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. 130 + 50 = 180 continue to add numbers mentally, including:

- twotwo-digit numbers
- three or more one-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

add numbers with up to three digits, using the formal written method of column addition

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to: solve problems, including missing number problems, using number facts, place value, and more complex addition

#### **Mental strategies**

Use of models and images:

Trios 7 + 5 = 12 5+7=12 12-5=7 12-7=5



- the empty number line

### To multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
00	210	220	230	240	250	260	270	280	290	300

Recognise and use the inverse relationship between addition and subtraction Use know ledge that addition can be done in any order (commutative), e.g.

- put the larger number first and count on in steps of 1, 10 or 100
- partition additions into hundreds, tens and ones, then recombine, e.g. 75 + 56 = 75 + 50 + 6
  - = 125+6
  - = 131

Identify near doubles, using doubles already know n, e.g. 70 + 71 Add the nearest multiple of 10 or 100, and adjust

Use patterns of similar calculations, e.g. 13 + 5 = 18 and 130 + 50 = 180

Use know ledge of the associative law when adding more than two numbers, e.g. 4 + 7 + 6 = (4 + 6) + 7= 10+7 = 17

#### Written methods

Add numbers with up to three digits (HTO + HTO) Estimate and check the answer to a calculation

#### Expanded written method

548+387

548
<u>+ 387</u>
15
120
800
935

The first stage in the written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. To find the partial sums either the ones or the hundreds can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method of columnar addition.

The addition of the tens in the calculation 548 + 387 is described in the words 'forty add eighty equals one hundred and tw enty', stressing the link to the related fact 'four add eight equals tw elve'. The addition of the hundreds is described as 'five hundred add three hundred equals eight hundred', stressing the link to the related fact 'five add three equals eight'.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

#### Formal written method of column addition

548+387

548 <u>+ 387</u> 935

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not ' carry one'.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

# SUBTRACTION

#### Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

continue to recall and use subtraction facts to 20 fluently, and derive and use related facts, e.g. 120 - 70 = 50 continue to subtract numbers mentally, including:

- twotwo-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

subtract numbers with up to three digits, using the formal written method of columnar subtraction estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to: solve problems, including missing number problems, using number facts, place value, and more complex subtraction



# MULTIPLICATION

 $Conceptual\ understanding\ and\ proce\ dural\ fluency$ 



consolidate recall of multiplication facts for the 2, 5 and 10 multiplication tables

recall and use multiplication facts for the 3, 4 and 8 multiplication tables

use know n multiplication facts to derive related facts involving multiples of 10, e.g.  $2 \times 30 = 60$ w rite and calculate mathematical statements for multiplication using the multiplication tables that they know, including for

two-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to: solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

12

48

#### **Mental strategies**



Make connections betw een arrays, number patterns and counting in steps of a constant size Understand and use the inverse relationship betw een multiplication and division Use doubling, e.g. connect the 2, 4 and 8 multiplication tables Use the 'key multiplication facts' of x 1, x 2, x 5, and x 10 to work out the answers to unknow n multiplication facts, e.g.  $7 \times 4 = (5 \times 4) + (2 \times 4)$ 

= 28

Use patterns of similar calculations, e.g.  $8 \times 6 = 48$  and  $8 \times 60 = 480$ Show that multiplication of two numbers can be done in any order (commutative), e.g.  $4 \times 12 \times 5 = 4 \times 5 \times 12$  $= 20 \times 12$ 

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit number by a one-digit number,  $63\times8=(60\times8)+(3\times8)$ 

= 240

= 480 + 24 = 504

#### Written methods

Short multiplication:

- Multiply a two-digit number by a one-digit number (TO x O) Estimate and check the answer to a calculation



The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in  $63 \times 8$  it is 'sixty multiplied by eight', not 'six multiplied by eight', although the relationship  $6 \times 8$  should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

#### Formal written method of short multiplication

×	63 28
	504

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

### DIVISION

### Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

consolidate recall of division facts for the 2, 5 and 10 multiplication tables

recall and use division facts for the 3, 4 and 8 multiplication tables

use know n division facts to derive related facts involving multiples of 10, e.g.  $60 \div 3 = 20$ 

w rite and calculate mathematical statements for division using the multiplication tables that they know develop reliable w ritten methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal w ritten method of short division (w ithout a remainder)

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to: solve problems, including missing number problems, involving division

#### **Mental strategies**



Make connections betw een arrays, number patterns and counting in steps of a constant size Understand and use the inverse relationship betw een multiplication and division

Use halving, e.g. find quarters by halving halves

Use patterns of similar calculations, e.g.  $48 \div 8 = 6$  and  $480 \div 80 = 6$ 

Understand and use the distributive law, e.g. partitioning when dividing a two-digit number by a one-digit number,

92÷4=(80÷4)+(12÷4)

= 20+3

= 23

#### Written methods

Short division (without a remainder):

- Divide a two-digit number by a one-digit number (TO  $\div$  O) Estimate and check the answer to a calculation

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answer is a multiple of 10? (20) There are 20 fours or 80, with 12 remaining. How many fours in 12? (3) So 92 divided by four is 23.'

#### Formal written method of short division



The expanded written method leads children to the formal written method of short division so that children fully

understand the procedure, and the effectiveness and efficiency of the method.

The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90. It is written in front of the 2 to show that 12 now has to be divided by 4.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

# Year 4

# NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

read and write numbers up to 10 000 in numerals and in words

count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards

recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

identify, represent and estimate numbers using different representations

find 10, 100 or 1000 more or less than a given number

compare and order numbers beyond 1000

round any number to the nearest 10, 100 or 1000

# DECIMALS

To add and subtract successfully, pupils need to:

recognise and write decimal equivalents of any number of tenths or hundredths

recognise the place value of each digit in a decimal to two decimal places

compare and order numbers with the same number of decimal places up to two decimal places

round decimals with one decimal place to the nearest whole number

understand the effect of multiplying and dividing a one-digit or two-digit number by 10 and 100

# ADDITION

#### Conceptual understanding and procedural fluency

To add successfully, pupils need to:

consolidate recall of addition facts to 20 and related facts involving multiples of 100 and 1000, e.g. 1300 + 500 = 1800 and 500 + 1300 = 1800

continue to add numbers mentally, including:

- twotwo-digit numbers
- three or more one-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

add numbers with up to four digits using the formal written method of columnar addition where appropriate, including calculations involving money, e.g.  $\pounds$ 13.56 +  $\pounds$ 38.54

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

solve addition two-step problems in contexts, deciding which operations and methods to use and why solve simple measure and money problems involving decimals to two decimal places

#### Mental strategies



If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

# SUBTRACTION

#### Conceptual understanding and procedural fluency

#### To subtract successfully, pupils need to:

consolidate recall of subtraction facts to 20 and related facts involving multiples of 100 and 1000,

e.g. 1800 - 500 = 1300 and 1800 - 1300 = 500

continue to subtract numbers mentally, including:

- twotwo-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate, including calculations involving money, e.g.  $\pounds 24.26 - \pounds 17.58$ 

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

solve subtraction two-step problems in contexts, deciding which operations and methods to use and why solve simple measure and money problems involving decimals to two decimal places

#### **Mental strategies**

- trios

Continue to use models and images when necessary:



- multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

Continue to use the relationship between addition and subtraction

Calculate mentally a difference such as 5005 - 2998 by counting up from the smaller to the larger number

Subtract the nearest multiple of 10, 100 or 1000, and adjust

Use patterns of similar calculations, e.g. 18 - 5 = 13 and 1800 - 500 = 1300

Use partitioning, e.g. 456 - 84 = 456 - 80 - 4

#### Written methods

Subtract numbers with up to four digits, including money and measures (ThHTO - ThHTO) Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

6418 – 2546

<sup>51311</sup> 6418 – 2546 <u>3872</u>

Start by subtracting the least significant digits first, i.e. the ones, then the tens, then the hundreds and finally the thousands. Refer to subtracting the tens, for example, by saying '11 tens subtract four tens', not '11 subtract four'. In this example the tens and the hundreds to be subtracted are larger than both the tens and hundreds you are subtracting from.

The calculation begins 8 subtract 6.

Then you exchange one of the 4 hundreds for 10 tens, crossing out 4 and writing a superscript 3, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 tens subtract 4 tens.

You then exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and writing a superscript 1 in front of the 3 to make 13 hundreds. The calculation then becomes 13 hundreds subtract 5 hundreds.

Then finally 5000 subtract 2000.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

# MULTIPLICATION

#### Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

consolidate recall of multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables

recall and use multiplication facts for the 6, 7, 9, 11 and 12 multiplication tables

use know n multiplication facts to derive related facts involving multiples of 10 and 100, e.g.  $200 \times 3 = 600$ 

use place value, know n and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers

recognise and use factor pairs and commutativity in mental calculations

multiply two-digit and three-digit numbers by a one-digit number using formal written layout

estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to: solve problems involving multiplying and adding, including using the distributive law and multiply two-digit numbers by one digit, including scaling problems and harder correspondence problems such as n objects are connected to m objects

#### **Mental strategies**



356×7=(300×7)+(50×7)+(6×7)

```
= 2100 + 350 + 42
= 2492
```

#### Written methods

#### Short multiplication:

- Multiply a two-digit or three-digit number by a one-digit number (TO  $\times$  O/HTO  $\times$  O) Estimate and check the answer to a calculation

Expanded written method

356	×7			356×7	
×	300	50	6	356	
7	2100	350	42= 2492	× 7 42 3 5 0 2100	( 6× 7) ( 50× 7) (300 × 7)
				2492	

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in  $356 \times 7$  it is 'fifty multiplied by seven', not 'five multiplied by seven', although the relationship  $5 \times 7$  should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

356×7

- 356 <u>× 347</u>
- 2492

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

# DIVISION

#### Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

consolidate recall of division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables

recall and use division facts for the 6, 7, 9, 11 and 12 multiplication tables

use known division facts to derive related facts involving multiples of 10 and 100, e.g.  $600 \div 3 = 200$ 

use place value, known and derived facts to divide mentally, including dividing by 1

recognise and use factor pairs in mental calculations

divide two-digit and three-digit numbers by a one-digit number using formal written layout (without a remainder) estimate and check the answer to a calculation, including using the inverse operation

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to: solve problems involving division in contexts, deciding which operations and methods to use and why

#### **Mental strategies**

Continue to use models and images:



Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,  $486 \div 9 = (450 \div 9) + (36 \div 9)$ 

### = 50+4

= 54

#### Written methods

Short division (without a remainder):

- Divide a two-digit or three-digit number by a one-digit number (TO  $\div$  O/HTO  $\div$  O) Estimate and check the answer to a calculation

#### Expanded written method

486÷9

 $\begin{array}{r}
 \frac{5 4}{9)4 8 6} \\
 4 5 0 (50 \times 9) \\
 \frac{3 6}{0} \\
 \hline
 (4 \times 9)
\end{array}$ 

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many nines divide into 480 so that the answer is a multiple of 10? (50) There are 50 nines or 450, with 36 remaining. How many nines in 36? (4) So 486 divided by nine is 54.'

#### Formal written method of short division

5 4 9)4 8<sup>3</sup>6

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 3 represents the 3 tens that are remaining after 9 has been divided into 480. It is written in front of the 6 to show that 36 now has to be divided by 9.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.



NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards

find 10, 100, 1000, 10 000 or 100 000 more or less than a given number

round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

# DECIMALS

To add, subtract, multiply and divide successfully, pupils need to: read, write, order and compare numbers with up to three decimal places identify the value of each digit in numbers given to three decimal places

multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents round decimals with two decimal places to the nearest whole number and to one decimal place

### ADDITION

#### Conceptual understanding and procedural fluency

To add successfully, pupils need to:

add numbers mentally with increasingly large numbers

add decimals, including a mix of one-digit whole numbers and tenths, e.g.  $1\cdot 3 + 0\cdot 5$ ; and complements of 1, e.g.  $0\cdot 83 + 0\cdot 17$ 

add whole numbers with more than four digits, including using the formal written method (columnar addition) add decimals to two places, including using the formal written method (columnar addition)

use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving number up to three decimal places

#### **Mental strategies**



# SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

subtract numbers mentally with increasingly large numbers

subtract decimals, including a mix of one-digit whole numbers and tenths, e.g. 1.8 - 0.7

subtract whole numbers with more than four digits, including using the formal written method (columnar subtraction) subtract decimals to two places, including using the formal written method (columnar subtraction) use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division and a combination of these, including

understanding the meaning of the equals sign

solve problems involving numbers up to three decimal places

#### **Mental strategies**

Continue to use models and images when necessary:

trios



- decimals addition and subtraction tables

	+	0	0-1	0.2	0-3	0.4	0.5	0-6	0.7	0.8	0.9	1
	0	0	0.1	0.2	0.3	0.4	0.5	0-6	0.7	0.8	0-9	1
	0.1	0.1	0-2	0.3	0.4	0.5	0.6	0-7	0.8	0.9	1	1-1
ĺ	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1-1	1.2
ĺ	0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3
ĺ	0.4	0.4	0.5	0.6	0-7	0.8	0.9	1	1.1	1.2	1.3	1.4
	0.5	0.5	0.6	0.7	0.8	0.9	1	1-1	1.2	1.3	1.4	1.5
	0.6	0.6	0.7	0.8	0-9	1	1.1	1.2	1.3	1.4	1.5	1.6
	0.7	0.7	0.8	0.9	1	1-1	1.2	1.3	1.4	1.5	1.6	1.7
ł	0.8	0.8	0.9	1	1-1	1.2	1.3	1-4	1-5	1.6	1.7	1.8
Í	0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1-8	1.9
Í	1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0.01	0·02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0-02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11
0.02	0.02	0.03	0-04	0.05	0.06	0.07	0.08	0.09	0-1	1.11	1.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1-11	1.12	1.13
0.04	0.04	0.05	0-06	0.07	0-08	0.09	0-1	1-11	1.12	1.13	1.14
0-05	0.05	0.06	0.07	0.08	0-09	0.1	1-11	1.12	1.13	1.14	1-15
0.06	0.06	0.07	0-08	0.09	0.1	1.11	1.12	1-13	1-14	1.15	1-16
0-07	0.07	0.08	0.09	0-1	1-11	1.12	1.13	1-14	1.15	1.16	1.17
0.08	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1-15	1.16	1.17	1.18
0-09	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19
0.1	0.1	1-11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	0.2

Develop further the relationship between addition and subtraction

Calculate mentally a difference such as 12 462 – 2300 by counting up from the smaller to the larger number Subtract the nearest multiple of 10, 100 or 1000, and adjust

Use patterns of similar calculations, e.g. 16 - 9 = 7 and  $1 \cdot 6 - 0 \cdot 9 = 0 \cdot 7$ 

Use partitioning, e.g. 456 - 84 = 456 - 80 - 4

```
= 372
```

#### Written methods

Subtract w hole numbers w ith more than four digits Subtract decimals w ith up to two decimal places Estimate and check the answ er to a calculation

#### Formal written method of columnar subtraction (decomposition)

45 257 – 17 488	83.72 - 36.4
314111417	7 13 6 12
45257	83.72
- 17488	- 36.49
27769	47-23

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying '14 tens subtract 8 tens', not '14 subtract 8'.

In the first example the ones, tens, hundreds and thousands to be subtracted are all larger than all of the ones, tens, hundreds and thousands you are subtracting from.

The calculation begins by exchanging one of the 5 tens for 10 ones, crossing out the 5 and writing a superscript 4, and crossing out the 7 and writing a superscript 17. The calculation then becomes 17 subtract 8.

You then exchange one of the 2 hundreds for 10 tens, crossing out the 2 and writing a superscript 1, and writing a superscript 1 in front of the 4 to make 14 tens. The calculation then becomes 14 tens subtract 8 tens.

Next, you exchange one of the 5 thousands for 10 hundreds, crossing out the 5 and writing a superscript 4, and writing a superscript 1 in front of the 1 to make 11 hundreds. The calculation then becomes 11 hundreds subtract 4 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 4 to make 14 thousands. The calculation then becomes 14 thousands subtract 7 thousands.

Then finally 30 000 subtract 10 000.

Where appropriate, place value columns are labelled, e.g. TO th, to remind children of the value of each of the digits.

# MULTIPLICATION

#### Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

consolidate recall of the multiplication facts for multiplication tables up to  $12 \times 12$ 

use know n multiplication facts to derive related facts involving multiples of 10, 100 and 1000, e.g.  $70 \times 80 = 5600$  continue to use place value, know n and derived facts to multiply mentally

multiply whole numbers and those involving decimals by 10, 100 and 1000

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish w hether a number up to 100 is prime and recall prime numbers up to 19

recognise and use square numbers and cube numbers, and the notation for squared  $\binom{2}{}$  and cubed  $\binom{3}{}$  continue to recognise commutativity in mental calculations

multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

solve problems, involving multiplication including using their know ledge of factors and multiples, squares and cubes solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates

**Mental strategies** 



- multiplication square to 12 x 12/multiples of 10 multiplication square

×	2	3	4	5	6	7	8	٩	10	11	12	×	20	30	40	50	60	70	80	90	100	110	120
1	2	3	4	5	6	7	8	9	10	11	12	1	20	30	40	50	60	70	80	90	100	110	120
2	4	6	8	10	12	14	16	18	20	22	24	2	40	60	80	100	120	140	160	180	200	220	240
3	6	9	12	15	18	21	24	27	30	33	36	3	60	90	120	150	180	210	240	270	300	330	360
4	8	12	16	20	24	28	32	36	40	44	48	4	80	120	160	200	240	280	320	360	400	440	480
5	10	15	20	25	30	35	40	45	50	55	60	5	100	150	200	250	300	350	400	450	500	550	600
6	12	18	24	30	36	42	48	54	60	66	72	6	120	180	240	300	360	420	480	540	600	660	720
7	14	21	28	35	42	49	56	63	70	77	84	7	140	210	280	350	420	490	560	630	700	770	840
в	16	24	32	40	48	56	64	72	80	88	96	8	160	240	320	400	480	560	640	720	800	880	960
9	18	27	36	45	54	63	72	81	90	99	108	9	180	270	360	450	540	630	720	810	900	990	1080
0	20	30	40	50	60	70	80	90	100	110	120	10	200	300	400	500	600	700	800	900	1000	1100	1200
1	22	33	44	55	66	77	88	99	110	121	132	11	220	330	440	550	660	770	880	990	1100	1210	1320
2	24	36	48	60	72	84	96	108	120	132	144	12	240	360	480	600	720	840	960	1080	1200	1320	1440

Continue to use the inverse relationship between multiplication and division

Use related facts and doubling and halving:

- double or halve the most significant digit first
- to multiply by 25, multiply by 100 then divide by 4
- double one number and halve the other
- find the multiplication facts for the  $\times$  16 multiplication table by doubling the  $\times$  8 multiplication facts. Use closely related facts:
- multiply by 19 or 21 by multiplying by 20 and adjusting
- develop the x 14 multiplication table by adding facts from the x 12 and x 2 multiplication tables Use factors, e.g. 9 x 18 = 9 x 6 x 3

Use patterns of similar calculations, e.g.  $8 \times 6 = 48$  and  $8 \times 600 = 4800$ Understand and use the commutative law, e.g.  $14 \times 12 = (2 \times 7) \times 12$ 

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one digit number, or two-digit numbers, e.g.

378×4=(300×4)+(70×4)+(8×4)	$78 \times 34 = (78 \times 30) + (78 \times 4)$
= 1200 + 280 + 32	= 2340 + 312
= 1512	= 2652

Written methods

Shor	t multiplic	ation:			
E	- Multi stimate a	ply num nd checl	bers up to four digit k the answ er to a ca	s by a one-digit n alculation	umber (HTO × O/ThHTO × O)
Grid	method			Expanded writte	en method
378×	:4			378×4	
×	300	70	8	378	
4	1200	280	32= 1512	× 4	
				32	( 8 × 4)
				280	( 70 × 4)
				1200	(300× 4)
				1512	· ·

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in  $378 \times 4$  it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship  $7 \times 4$  should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

#### Formal written method of short multiplication

378×4

378 <u>×334</u> 1512

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Long multiplication:

- Multiply numbers up to three digits by a two-digit number (TO  $\times$  TO/HTO  $\times$  TO) Estimate and check the answer to a calculation

#### Grid method

78×44

				78	
×	70	8		× 34	
4	280	32	312→	312	(78 × 4)
30	2100	240	+ 2340 →	2340	(78 × 30)
			2652 →	2652	

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 78 is multiplied by 30 (using know ledge of 78 x 3), then 78 is multiplied by 4, and finally the two products are added together.

### Expanded written method

#### 78×34

Multiplying the most significant digit first

7 8 3 4  $23240 (78 \times 30)$   $3132 (78 \times 4)$  2652

Multiplying the least significant digit first 7 8 × 3 4

3 132 (78 × 4)

2652

2 324 0 (78 × 30)

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

### DIVISION

#### Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

consolidate recall of the division facts for multiplication tables up to 12  $\times$  12

use know n division facts to derive related facts involving multiples of 10, 100 and 1000, e.g.  $6300 \div 90 = 70$  continue to use place value, know n and derived facts to divide mentally

divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

establish whether a number up to 100 is prime and recall prime numbers up to 19

divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

solve problems involving division, including using their knowledge of factors and multiples, squares and cubes

solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

solve problems involving division, including scaling by simple fractions and problems involving simple rates

#### **Mental strategies**





- tric

12×7=84 84÷12=7

84÷7=12

multiplication square to 12 x 12/multiples of 10 multiplication square

×	2	3	4	5	6	7	8	9	10	11	12	×	20	30	40	50	60	70	80	90	100	110	120
1	2	3	4	5	6	7	8	9	10	11	12	1	20	30	40	50	60	70	80	90	100	110	120
2	4	6	8	10	12	14	16	18	20	22	24	2	40	60	80	100	120	140	160	180	200	220	240
3	6	9	12	15	18	21	24	27	30	33	36	3	60	90	120	150	180	210	240	270	300	330	360
4	8	12	16	20	24	28	32	36	40	44	48	4	80	120	160	200	240	280	320	360	400	440	480
5	10	15	20	25	30	35	40	45	50	55	60	5	100	150	200	250	300	350	400	450	500	550	600
6	12	18	24	30	36	42	48	54	60	66	72	6	120	180	240	300	360	420	480	540	600	660	720
7	14	21	28	35	42	49	56	63	70	77	84	7	140	210	280	350	420	490	560	630	700	770	840
8	16	24	32	40	48	56	64	72	80	88	96	8	160	240	320	400	480	560	640	720	800	880	960
9	18	27	36	45	54	63	72	81	90	99	108	٩	180	270	360	450	540	630	720	810	900	990	1080
10	20	30	40	50	60	70	80	90	100	110	120	10	200	300	400	500	600	700	800	900	1000	1100	1200
11	22	33	44	55	66	77	88	99	110	121	132	11	220	330	440	550	660	770	880	990	1100	1210	1320
12	24	36	48	60	72	84	96	108	120	132	144	12	240	360	480	600	720	840	960	1080	1200	1320	1440

Continue to use the inverse relatio	nship between multiplication a	nd division
Mental strategies continued		
Continue to use halving, e.g. conn Understand and use the distribut $486 \div 9 = (450 \div 9) + (36 \div 9)$ = 50+4 = 54	ect the 3, 6 and 12 multiplication utive law , e.g. partitioning when	n tables n dividing a three-digit number by a one-digit number,
Written methods		
Short division (including with rema	inders expressed as a w hole n	umber, fraction or decimal):
- Divide numbers up to 4 of Estimate and check the answe	digits by a one-digit number (HT r to a calculation	O ÷ O/ThHTO ÷ O)
Formal written method of short	division	
Whole number remainder	Fraction remainder	Decimal remainder
279÷6	279÷6	279÷6
4 6 r 3 6) 2 7 39	4 6 <sup>1</sup> / <sub>2</sub> 6) 2 7 <sup>3</sup> 9	<b>4</b> 16 <b>. 5</b> 3 3
Children should describe what the that the answer is a multiple of 1 tens that are remaining after 6 h divided by 6.	ey are doing using phrases sin 0? (40) There are 40 sixes or 2 as been divided into 270. It is	ilar to the following: 'How many sixes divide into 270 so '40, with 30 remaining.' The superscript 3 represents the 3 written in front of the 9 to show that 39 now has to be

Children then ask: 'How many sixes in 39?' (6 remainder 3). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or dow n.

# Year 6

# NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read, w rite, order and compare numbers up to 10 000 000 and determine the value of each digit count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards find 10, 100, 1000, 10 000 or 100 000 more or less than a given number round any w hole number to a required degree of accuracy

# DECIMALS

To add, subtract, multiply and divide successfully, pupils need to: read, write, order and compare numbers with up to three decimal places identify the value of each digit in numbers given to three decimal places multiply and divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents round decimals with two decimal places to the nearest whole number and to one decimal place

### **ADDITION**

#### Conceptual understanding and procedural fluency

#### To add successfully, pupils need to:

perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations practise addition for larger numbers and decimals, using the formal written method of columnar addition

use know ledge of the order of operations to carry out calculations involving the four operations

use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

solve addition multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division

use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

solve problems which require answers to be rounded to specified degrees of accuracy

#### **Mental strategies**

Continue to use models and images when necessary:

decimals addition and subtraction tables

+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	0.0	1 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11
0.2	0.2	0.3	0.4	0-5	0.6	0.7	0.8	0.9	1	1.1	1.2	0.0	2 0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	0.0	3 0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	0.0	4 0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14
0.5	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	0.0	5 0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15
0.6	0.6	0.7	0.8	0.9	1	1-1	1.2	1.3	1-4	1.5	1.6	0.0	6 0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16
0.7	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	0.0	7 0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	0.0	3 0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	0.0	9 0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	0.1	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	0.2

+	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11
0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12
0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13
0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14
0.05	0.05	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15
0.06	0.06	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16
0.07	0.07	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17
0.08	0.08	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18
0.09	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19
0.1	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19	0.2

Develop further the relationship between addition and subtraction Continue to apply know ledge of the commutative law, e.g.

put the larger number first and count on in steps of 1, 10, 100 or 1000

partition additions into hundreds, tens and ones, then recombine, e.g. 3356 + 257 = 3356 + 200 + 50 + 7

= 3556 + 57 = 3613

Identify near doubles, using doubles already know n, e.g. 5.7 + 5.8

Add the nearest multiple of 10, 100 or 1000, and adjust

Use patterns of similar calculations, e.g. 9 + 7 = 16 and 0.09 + 0.07 = 0.16

Use know ledge of the associative law when adding more than two numbers, e.g. 24 + 27 + 16 = (24 + 16) + 27

= 40 + 27= 67

#### Written methods

Add numbers with more than four digits

Add decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places

Estimate and check the answer to a calculation

#### Formal written method of columnar addition

456 287 + 359 849	57.486 + 45.378
456287	57-486
+359849	+ 45.378
816136	102-864
11111	1 1 1

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand', ... not 'carry one'. Where appropriate, place value columns are labelled, e.g. TO-tht<sup>h</sup>, to remind children of the value of each of the digits.

### SUBTRACTION

#### Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations practise subtraction for larger numbers and decimals, using the formal written method of columnar subtraction

use know ledge of the order of operations to carry out calculations involving the four operations

use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

solve problems involving addition, subtraction, multiplication and division

use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

solve problems which require answers to be rounded to specified degrees of accuracy

#### **Mental strategies**

Continue to use models and images when necessary:

decimals addition and subtraction tables

+	0	0-1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0-1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1-1	1.2
0.3	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1-2	1.3
0.4	0.4	0.5	0.6	0.7	0.8	0.9	1	1-1	1.2	1-3	1.4
0.5	0.5	0-6	0.7	0.8	0.9	1	1.1	1.2	1.3	1-4	1.5
0.6	0.6	0.7	0.8	0.9	1	1-1	1.2	1.3	1.4	1.5	1.6
0.7	0.7	0.8	0.9	1	1-1	1.2	1.3	1.4	1.5	1.6	1.7
0.8	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
0.9	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2

+	0	0-01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0-1
0	0	0-01	0-02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0-1
0.01	0.01	0-02	0.03	0.04	0-05	0.06	0.07	0.08	0.09	0-1	1.11
0.02	0.02	0-03	0.04	0.05	0-06	0.07	0-08	0.09	0.1	1-11	1.12
0·03	0.03	0-04	0.05	0.06	0.07	0.08	0-09	0-1	1-11	1.12	1.13
0-04	0.04	0.05	0.06	0.07	0-08	0.09	0.1	1.11	1.12	1.13	1.14
0.05	0.05	0-06	0.07	0.08	0.09	0.1	1-11	1.12	1.13	1.14	1.15
0.06	0.06	0-07	0.08	0.09	0-1	1.11	1.12	1-13	1.14	1.15	1-16
0.07	0.07	0-08	0.09	0.1	1-11	1.12	1.13	1-14	1.15	1.16	1.17
0.08	0.08	0-09	0.1	1.11	1-12	1.13	1.14	1.15	1.16	1.17	1.18
0.09	0.09	0.1	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	1.19
0.1	0.1	1-11	1.12	1.13	1-14	1.15	1.16	1.17	1.18	1.19	0.2

Develop further the relationship betw een addition and subtraction Calculate mentally a difference such as 23 004 - 18 998 by counting up from the smaller to the larger number

Subtract the nearest multiple of 10, 100 or 1000, and adjust

Use patterns of similar calculations, e.g. 16 - 9 = 7 and 0.16 - 0.09 = 0.07

Use partitioning, e.g. 4656 - 358 = 4656 - 300 - 50 - 8= 4356 - 58

= 4298

#### Written methods

Subtract numbers with more than four digits

Subtract decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places

Estimate and check the answer to a calculation

#### Formal written method of columnar subtraction (decomposition)

746 291 – 298 354 63.237 – 45.869

6131512811	512111217
746 291	63-237
-298 354	-45-869
447 937	17. 368

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens ... and finally the hundreds of thousands. Refer to subtracting the tens, for example, by saying '8 tens subtract 5 tens', not '8 subtract 5'.

In the first example, the ones, tens, thousands and tens of thousands to be subtracted are all larger than all of the ones, tens, thousands and tens of thousands you are subtracting from.

The calculation begins by exchanging one of the 9 tens for 10 ones, crossing out the 9 and writing a superscript 8, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 subtract 4.

You then calculate 8 tens subtract 5 tens.

Next, you exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 hundreds subtract 3 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 5 to make 15 thousands. The calculation then becomes 15 thousands subtract 8 thousands.

Next, you exchange one of the 7 hundreds of thousands for 10 tens of thousands, crossing out the 7 and writing a superscript 6, and writing a superscript 1 in front of the 3 to make 13 tens of thousands. The calculation then becomes 13 t ens of thousands subtract 9 tens of thousands.

Then, finally, 600 000 subtract 200 000.

Where appropriate, place value columns are labelled, e.g. TO thth, to remind children of the value of each of the digits.

### MULTIPLICATION

#### Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

consolidate recall of the multiplication facts for multiplication tables up to 12 x 12

use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, and decimals,

e.g. 70 × 80 = 5600,  $0.8 \times 6 = 4.8$ 

perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations continue to multiply whole numbers and those involving decimals by 10, 100 and 1000

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

establish whether a number up to 100 is prime and recall prime numbers up to 19

recognise and use square numbers and cube numbers, and the notation for squared  $(^2)$  and cubed  $(^3)$  continue to recognise commutativity in mental calculations

practise multiplication for larger numbers, using the formal written method of short multiplication

multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

multiply one-digit numbers with up to two decimal places by whole numbers

multiply numbers with up to two decimal places by one- and two-digit whole numbers

use know ledge of the order of operations to carry out calculations involving the four operations

use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to: solve problems involving addition, subtraction, multiplication and division

use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

solve problems which require answers to be rounded to specified degrees of accuracy

**Mental strategies** 

84÷12=7

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Continue to use models and images when necessary:



84÷7=12 /7 12

- multiplication square to 12 x 12/multiples of 10 multiplication square/decimals multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144
								•			
x	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2

1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	<mark>240</mark>
3	60	90	120	150	180	210	240	270	300	330	<mark>360</mark>
4	80	120	160	200	240	280	320	360	400	440	<mark>480</mark>
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

x 20 30 40 50 60 70 80 90 100 110 120

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6
4	0.8	1.2	1.6	2	2.4	2.8	3.2	3.6	4	4.4	4.8
5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
6	1.2	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6.6	7.2
7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7	7.7	8.4
8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8.8	9.6
9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.8
10	2	3	4	5	6	7	8	9	10	11	12
11	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2
12	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	13.2	14-4

Continue to use the inverse relationship between multiplication and division

Use related facts and doubling and halving:

- double or halve the most significant digit first
- to multiply by 25, multiply by 100 then divide by 4
- double one number and halve the other
- find the multiplication facts for the x 24 multiplication table by doubling the x 12 multiplication facts
- squares of multiples of 10 to 100, e.g. 70 × 70, and the corresponding halves
- doubles of decimals, e.g.  $4.7 \times 2$ ,  $0.63 \times 2$ , and the corresponding halves
- doubles of multiples of 10 to 1000, e.g. 830 x 2, and the corresponding halves
- doubles of multiples of 100 to 10 000, e.g. 48 500 × 2, and the corresponding halves

Use closely related facts: multiply by 49 or 51 by multiplying by 50 and adjusting develop the x 18 multiplication table by adding facts from the x 10 and x 8 multiplication tables Use factors, e.g.  $9 \times 18 = 9 \times 6 \times 3$ Use patterns of similar calculations, e.g.  $8 \times 6 = 48$  and  $0.8 \times 6 = 4.8$ Continue to use and apply the commutative law Understand and use the associative law, e.g.  $10.6 \times 30 = 10.6 \times (10 \times 3)$  or  $= (10.6 \times 10) \times 3$ Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, or two-digit numbers, and partitioning when multiplying a whole number or decimal by a one-digit number, e.g.  $285 \times 63 = (200 \times 63) + (80 \times 63) + (5 \times 63)$  $4 \cdot 83 \times 6 = (4 \times 6) + (0 \cdot 8 \times 6) + (0 \cdot 03 \times 6)$ = 12 600 + 5040 + 315  $= 24 + 4 \cdot 8 + 0 \cdot 18$ = 17 955 = 28.98Written methods Short multiplication (whole numbers): Multiply multi-digit numbers up to 4 digits by a one-digit whole number Estimate and check the answer to a calculation Formal written method of short multiplication 5643 × 8 5643 53 2**8** × 45144 Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the dig its. Short multiplication (Decimals): Multiply one-digit or two-digit numbers with up to two decimal places by a one-digit number Estimate and check the answer to a calculation Grid method Expanded written method 4.83 × 6  $4.83 \times 6$  is equivalent to  $483 \times 6 \div 100$ 4 0.8 0.03 4.83 x 4.8 6 24 0.18 = 28.986 18 ( 3 × 6) 480 (80 × 6) 2400 (480 × 6) 2898 2898 ÷ 100 = 28.98 The first step is to show all of the calculations involved. Children should describe what they do by referring to the actual values of the digits in the columns, e.g. when using the grid method and multiplying the tenths in 4.83 × 6 it is 'zero point eight multiplied by six', not 'eight multiplied by six when using the expanded written method and multiplying the tens in 483 × 6 it is 'eighty multiplied by six, not 'eight multiplied by six'. Although for both methods the relationship  $8 \times 6$  should be stressed. Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits. Formal written method of short multiplication  $4.83 \times 6$  is equivalent to  $483 \times 6 \div 100$ 483 × 416 2898

2898 ÷ 100 = 28-98

The expanded written method leads to the formal written method of short multiplication so that children fully understand the

procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Long multiplication (whole numbers):

Multiply multi-digit numbers up to four digits by a two-digit number (TO x TO / HTO x TO) Estimate and check the answer to a calculation

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 285 is multiplied by 60 (using knowledge of 285 x 6), then 285 is multiplied by 3, and finally the two products are added together.

#### Expanded written method

Multiplying the most significant digit first

olying the n	nost significant digit f	irst	Multi	plying t	he least significant digit first
285				285	
63			×	63	
7 <sup>5</sup> 1 <sup>3</sup> 0 0 8 <sup>2</sup> 5 <sup>1</sup> 5	(285 × 60) (285 × 3)			8 <sup>2</sup> 5 <sup>1</sup> 5 17 <sup>5</sup> 1 <sup>3</sup>	(285 × 3) 300(285×60)
17955	, , , , , , , , , , , , , , , , , , ,			17955	, , , , , , , , , , , , , , , , , , ,

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method\_will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

### Formal written method of long multiplication

285 x 63

× 1

285 × 63 8<sup>2</sup>5<sup>1</sup>5  $17^{5}1^{3}00$ 17955

The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Long multiplication (Decimals):

Multiply one-digit numbers with up to two decimal places by a two-digit number Estimate and check the answer to a calculation

The first step is to show all of the calculations involved, e.g. 7.56 is multiplied by 30 (using know ledge of 756 x 3), then 7.56 is multiplied by 4, and finally the two products are added together.

1

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals Expanded written method

Multiplying the most significant digit first	Multiplying the least significant digit first
7.56 × 34 is equivalent to 756 × 34 ÷ 100	7.56 × 34 is equivalent to 756 × 34 ÷ 100
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$756 \times 34$ $30^{2}2^{2}4  (756 \times 4)$ $22^{1}6^{1}80  (756 \times 30)$ $25704$
25 704 ÷ 100 = 257·04	25 704 ÷ 100 = 257·04
Where appropriate, place value columns are labelled, e.g. HT	D, to remind children of the value of each of the digits.
Formal written method of long multiplication	
7.56 × 34 is equivalent to 756 × 34 ÷ 100	
$756$ $x 34$ $30^{2}2^{2}4$ $22^{1}6^{1}80$ $25704$ 1	
25 704 ÷ 100 = 257·04	
The expanded written method leads to the formal written meth procedure, and the effectiveness and efficiency of the method.	od of long multiplication so that children fully understand the
The amount of time that should be spent teaching and practisi children are in their recall of number facts and in their underst	ing the expanded written method will depend on how secure the anding of place value.
Where appropriate, place value columns are labelled, e.g. HT	D, to remind children of the value of each of the digits.

# DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

consolidate recall of the division facts for multiplication tables up to  $12 \times 12$ 

use know n division facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g.  $6300 \div 90 = 70$ ,  $6\cdot 3 \div 9=0\cdot 7$ 

perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations continue to divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places

identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

establish whether a number up to 100 is prime and recall prime numbers up to 19

practise division for larger number, using the formal written method of short division

divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

divide numbers with up to two decimal places by one- and two-digit whole numbers

use know ledge of the order of operations to carry out calculations involving the four operations

use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

#### Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

solve problems involving addition, subtraction, multiplication and division

use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

solve problems which require answers to be rounded to specified degrees of accuracy

#### **Mental strategies**

Continue to use models and images when necessary:

number lines	1	2	3	4	5	6	7	8	9	10	11	12
	12	24	36	48	60	72	84	96	108	120	132	144

- multiplication square to 12 x 12/multiples of 10 multiplication square/decimals multiplication square

	Z	3	4	5	0	/	8	Ч	10	11	12	×	20	30	40	50	60	70	80	40	100	110	120
1	2	3	4	5	6	1	8	9	10	11	12	1	20	30	40	50	60	10	80	40	100	110	120
4	4	6	8	10	12	14	16	18	20	22	24	2	40	60	80	100	120	140	160	180	200	220	240
3	6	4	12	15	18	21	24	21	30	33	36	3	60	90	120	150	180	210	240	2/0	300	330	360
4	10	12	10	20	24	20	32	30	40	44 EE	48	4	100	120	200	200	240	260	320	360 6E0	400	440 550	480
5	12	12	20	20	30	33	40	40 54	60	55	72	5	100	180	200	300	360	/20	400	5/0	600	660	720
7	14	21	28	35	42	49	56	63	70	77	84	7	140	210	280	350	420	490	560	630	700	770	840
8	16	24	32	40	48	56	64	72	80	88	96	8	160	240	320	400	480	560	640	720	800	880	960
9	18	27	36	45	54	63	72	81	90	99	108	9	180	270	360	450	540	630	720	810	900	990	1080
10	20	30	40	50	60	70	80	90	100	110	120	10	200	300	400	500	600	700	800	900	1000	1100	1200
11	22	33	44	55	66	77	88	99	110	121	132	11	220	330	440	550	660	770	880	990	1100	1210	1320
12	24	36	48	60	72	84	96	108	120	132	144	12	240	360	480	600	720	840	960	1080	1200	1320	1440
						_	_		_														
×	0.2	0.3	0-4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2												
1	0.2	0.3	0.4	0.5	0-6	0.7	0.8	0.9	1	1.1	1.2												
2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4												
3	0.6	0-9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3	3.6												
4	0.8	1.2	1.6	2	2.4	2.8	3.2	3-6	4	4.4	4.8												
5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6												
0	1-2	2.1	2.4	3	3.0	4.2	4.0	5.4	0	7.7	9.4												
8	1.6	2.4	3.2	4	4.2	5.6	6.4	7.2	8	8.8	9.6												
9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	9.9	10.9												
10	2	3	4	5	6	7	8	9	10	11	12												
11	2.2	3.3	4-4	5.5	6.6	7.7	8.8	9.9	11	12.1	13.2												
-	2.4	3.6	4.8	6	7.2	8.4	9.6	10-8	12	13.2	14-4												

= 54

### Written methods

Short division, including with remainders express	sed as a whole number, fra	action or decimal (whole numbers)
<ul> <li>Divide numbers up to 4 digits by a one- Estimate and check the answ er to a calculation</li> </ul>	digit number (HTO ÷O/Th n	HTO ÷O)
Formal written method of short division		
Whole number remainder	Fraction remainder	Decimal remainder
1838÷8	1838÷8	1838÷8
2 2 9 r 6 8) 1 8 <sup>27</sup> 3 8	229 <sup>2</sup> 8)1838	
Children should describe what they are doing us the answer is a multiple of 100?' (200) There are 2 hundreds that are remaining after 8 has been of (230) now have to be divided by eight. Children then ask: 'How many eights divide into 2	ing phrases similar to the f 200 eights or 1600, with 2 divided into 1800. It is writt 230 so that the answer is a	follow ing: 'How many eights divide into 1800 so that 200 remaining. The superscript 2 represents the en in front of the 3 to show that a total of 23 tens a multiple of 10?' (20) There are 20 eights or 160, with 70
to show that 78 now has to be divided by 8.	ns that are remaining after	The sector the remainder is written as a whele
number, fraction, decimal or rounded up or dow r	emainder 6). Depending of n.	n the context, the remainder is written as a whole
Short division (Decimals) - Divide numbers with up to two decimal Estimate and check the answer to a calculatio	places by a one-digit num n	per (O·th ÷ O/TO·th ÷ O)
Method 1: Calculating with decimals		
7.5 £		
Method 2: Converting decimals to whole num 45.36 ÷ 6 is equivalent to 4536 ÷ 6 ÷ 100	nbers before calculating,	then converting the answer back to decimals
7,5,£ 6)4536		
$756 \div 100 = 7.56$ Phrases similar to those above for short division An emphasis should be placed on recognising th	of wholenumbers should le value of each of the digi	be used for short division involving decimals. is in the dividend.
Long division, including with remainders express - Divide numbers up to 4 digits by a two	ed as a w hole number, fra digit number (HTO ÷ TO/TI	nction or decimal (Whole numbers) nHTO ÷TO)
Expanded written method of long division	Formal writ	ten method of long division
$\begin{array}{r} 3 \ 2 \ 4 \ r \ 4 \\ 18 \hline 5 \ 8 \ 3 \ 6 \\ ^{45_3} \ 4 \ 0 \ 0 \\ - \ 3 \ 6 \ 0 \\ - \ 0 \ 7 \ 2 \\ ^{4} \\ 4 \end{array} $	18) 5 8 - 5	3 2 4 r 4 3 3 6 4 4 v 3 3 ₹
$5836 \div 18 = 324 \text{ r } 4 \text{ or } \frac{1}{3}$	5836 ÷ 18 = 324 r 4 or	

The amount of time that should be spent teaching and practising the expanded written method of long division will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.

Long division (Decimals)

- Divide numbers with up to two decimal places by a two-digit whole number (TO th  $\div$  TO) Estimate and check the answer to a calculation

#### Method 1: Calculating with decimals

Expanded written method of long division	Formal written method of long division
58·32 ÷ 18	58·32 ÷ 18
$ \begin{array}{r} 3 & -2 & 4 \\ 18) & 5 & 8 & -3 & 2 \\  & 5 & 4 & -0 & 0 \\  & 4 & -3(0.2 \times 18) \\  & - & 3 & -6 & 0 \\  & & 7 \\ \end{array} $	$   \begin{array}{r}     3 - 2 4 \\     18) 5 8 - 3 2 \\     - 5 4 - 3 \\     \overline{4} + 3 \\     \underline{3 - 6}   \end{array} $
- <u>0 7 2</u>	
0 - 0 0	

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e Method 1) will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting whole and decimal numbers mentally, and in their understanding of place value.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

#### Expanded written method of long division

58.32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

58-32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

Formal written method of long division

324 18)5832 5×400	(300 × 18)	324 18)5832 - 54
432 - <u>360</u> 7	(20×18)	4v3 3€
- 072		- 72