## Thornhill Primary School Policy for Calculation

The policy was developed by the Curriculum Leader for Maths and reviewed by staff and SLT and agreed in September 2020.

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new National Curriculum. Through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. Children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at ageappropriate levels as set out in the National Curriculum 2014 and in line with school policy.


#### Abstract

Aims: The aim of the calculation policy is to ensure consistency throughout the school and to make teachers aware of the continuity and progression in skill development across the year groups. It enables staff, and other stakeholders where relevant, to see how the concepts, facts and calculation strategies are applied and taught in each year group. This is to enable children to calculate successfully and be able to: have a confident and competent understanding of numbers and the number system have instant recall of a set of basic number facts (fluency) use a range of mental calculation strategies effectively, efficiently and appropriately use and apply all of the above to solve problems and reason mathematically.


The above aims are linked to the Mathematics policy aims which are:
To develop knowledge and understanding of mathematical ideas, processes and skills and relate these to everyday experiences.
To acquire and refine mathematical and practical skills necessary to investigate ideas and questions.
To develop a progressive understanding of mathematical concepts, skills and attitudes appropriate to all children's abilities.
To promote mathematics across the curriculum where appropriate and where it enhances mathematical application.

Children deserve:
To be set appropriate learning challenges.
To be taught well and be given the opportunity to learn in ways that maximise the chances of success. To have support to enable them to tackle the specific barriers to progress they face.

## Objectives:

We will achieve these aims by the following objectives:
To provide pupils with the opportunity to develop their mathematical skills to the best of their ability. To develop their skills and potential within a broad curriculum to include practical tasks, problem solving and real life situations.
To develop the ability to select and use appropriate mathematical equipment and materials.
To create an enthusiastic and positive attitude towards the learning and application of the subject.

## Organisation

## Teaching \& Learning

## The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to $9+9$ and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. $5+8+4$ )
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. $600+700,160-70$ )
- partition 2 and 3 digit numbers into multiples of 100,10 and 1 in different ways (e.g. partition 74 into $70+4$ or $60+14$ )
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to $12 \times 12=144$ and division facts to $144 \div 12=12$
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10,100 and later 1000
- recognise factor pairs of numbers (e.g. that $15=3 \times 5$, or that $40=10 \times 4$ ) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into $100 \mathrm{~s}, 10 \mathrm{~s}$ and 1 s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where $n$ objects are related to $m$ objects
- investigate and learn rules for divisibility

See the following pages for pedagogy and specific examples of progression.

## Appendix

## Year 1

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:
count, read and w rite numbers from 1 to 20 in numerals and words
count, read and w rite numbers to 100 in numerals
count to and across 100, form ards and back ards, beginning with 0 or 1, or from any given number count in multiples of twos, fives and tens given a number, identify one more and one less compare and order numbers to at least 20
identify and represent numbers using objects and pictorial representations, including the number line, and use the language of: equal to, more than, less than (few er), most, least

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
understand addition as combining two or more groups of objects
understand addition as counting on
represent and use number bonds within 20
add one-digit and two-digit numbers to 20 , including zero
realise the effect of adding zero
recall doubles of all numbers to 10
understand that addition can be done in any order
read, w rite and interpret mathematical statements involving addition (+) and equals (=) signs

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing
number problems such as $16=\square \square 7$
solve one-step problems that involve addition in familiar contexts, e.g. money

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations

- number tracks and number lines

- 1-100 number square

| 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Use of resources such as Numicon
Identify near doubles, using doubles already known (e.g. $6+5$ )
Recognise and use patterns of similar calculations (e.g. $10+0=10,9+1=10,8+2=10 \ldots$ )
Understand and use the inverse relationship betw een addition and subtraction

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
understand subtraction as 'taking aw ay' (counting back)
understand subtraction as 'finding the difference' (counting up)
represent and use subtraction facts w ithin 20
subtract one-digit and tw o-digit numbers to 20 , including zero
realise the effect of subtracting zero
understand that subtraction cannot be done in any order
read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
solve one-step problems that involve subtraction, using concrete objects and pictorial repres entations, and missing number problems such as $7=\square-9$
solve one-step problems that involve subtraction in familiar contexts, e.g. money

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations

-     - number tracks and number lines: 'take aw ay' (counting back)

'finding the difference' (counting
up)


Use of resources such as Numicon

1-100 number square

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |  |

Recognise and use patterns of similar calculations
(e.g. $10-0=10,10-1=9,10-2=8$ )

Understand and use the inverse relationship betw een addition and subtraction

## M ULTIPLICATION

## Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
understand multiplication through grouping small quantities
understand the link betw een multiplication and doubling

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:
solve one-step problems involving multiplication, by calculating the answ er using concrete objects, pictorial representations and arrays w ith the support of the teacher
solve one-step problems that involve multiplication in familiar contexts

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations

arrays


4 lots of 5 is 20
5 lots of 4 is 20

Make connections betw een arrays, number patterns and counting in steps of a constant size

## DIVISION

## Conceptual understanding and procedural fluency

To divide successfully, pupils need to:
understand division through sharing small quantities understand the link betw een division and halving

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to: solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays $w$ ith the support of the teacher
solve one-step problems that involve division in familiar contexts

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations


Make connections betw een arrays, number patterns and counting in steps of a constant size


## Year 2

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read and w rite numbers to at least 100 in numerals and in words
count in steps of 2,3 , and 5 from 0 , and in tens from any number, forw ards and backw ards
recognise the place value of each digit in a tw o-digit number (tens, ones)
identify, represent and estimate numbers using different representations, including the number line
compare and order numbers from 0 up to 100; use <, > and = signs

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
recall and use addition facts to 20 fluently, and derive and use related facts up to 100, including adding tw o multiples of 10 ,
e.g. $30+50$
add numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- three one-digit numbers
show that addition of tw o numbers can be done in any order (commutative) and subtraction of one number from another cannot
recognise and use the inverse relationship betw een addition and subtraction and use this to check calculations and solve missing number problems
record addition in columns to support place value and prepare for the formal written method with larger numbers


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve problems with addition:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing know ledge of mental and written methods


## Mental strategies

Use of models and images:

- concrete objects/pictorial representations

- number tracks and number lines


1-100 number square

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- trios

- empty number line


Use know ledge that addition can be done in any order (commutative), e.g.

- put the larger number first and count on in tens or ones
- add three small numbers by putting the largest number firstand/or find a pair totalling 10

Partition additions into tens and ones, then recombine, e.g.

$$
38+25=30+20+8+5
$$

$$
=50+13
$$

$$
=63
$$

$38+25=38+20+5$
$=58+5$
$=63$

38+25

$$
\begin{array}{r}
30+8 \\
+\quad 20+5 \\
\hline
\end{array} \quad 50+13=63
$$

Identify near doubles, using doubles already known (e.g. $7+8,30+31$ )
Add a 'near multiple of 10 ' to a tw o-digit number by adding $10,20,30$ and adjusting
Recognise and use patterns of similar calculations (e.g. $10+0=10,9+1=10,8+2=10 \ldots$ )
Understand and use the inverse relationship betw een addition and subtraction

Add tw o tw o-digit numbers: TO + TO (where answ ers do not exceed 100)

## Expanded written method

38+25

| 38 |
| ---: |
| $+\frac{25}{13}$ |
| 50 |
| 63 |

Record addition calculations in columns to support place value and prepare for the formal written method of columnar addition with larger numbers.
The first stage in the written method shows separately the addition of the ones to the ones and the tens to the tens. To find the partial sums either the ones or the tens can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal w ritten method.
The addition of the tens in the calculation $38+25$ is described in the words 'thirty add tw enty equals fifty', stressing the link to the related fact three add tw o equals five'.
Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100, including subtracting two multiples of 10 , e.g. $80-30$
subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
show that addition of tw o numbers can be done in any order (commutative) and subtraction of one number from another cannot
recognise and use the inverse relationship betw een addition and subtraction and use this to check calculations and solve missing number problems
record subtraction in columns to support place value and prepare for the formal written method with larger numbers


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
solve problems w ith subtraction:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing know ledge of mental and w ritten methods


## Mental strategies

Use of models and images:
concrete objects/pictorial representations


- number tracks and number lines: 'take aw ay' (counting back)
'finding the difference' (counting up)
- 1-100 number square

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- trios

- empty number line: 'take aw ay' (counting back)

'finding the difference' (counting up)


Find a small difference by counting up from the smaller to the larger number, e.g. $51-4$
Subtract a 'near multiple of 10 ' from a tw o-digit number by subtracting 10, 20, 30 and adjusting
Recognise and use patterns of similar calculations (e.g. $10-0=10,10-1=9,10-2=8 \ldots$ )
Understand and use the inverse relationship betw een addition and subtraction
Use partitioning, e.g.

$$
\begin{aligned}
& 52-28=52-20-8 \\
& =32-8 \\
& =24
\end{aligned}
$$

## Written methods

Subtract tw o tw o-digit numbers: TO - TO (that do not require decomposition)
87-32

$$
\begin{array}{r}
87 \\
-32 \\
\hline 55 \\
\hline
\end{array}
$$

Record subtraction calculations that do not require decomposition in columns to support place value and prepare for formal w ritten methods of columnar subtraction with larger numbers.
Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

## Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
recognise multiplication as repeated addition
recall and use multiplication facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers
show that multiplication of tw o numbers can be done in any order (commutative) and division of one number by another cannot
calculate mathematical statements for multiplication within the multiplication tables and w rite them using the multiplication ( $x$ ) and equals (=) signs

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:
solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations
- arrays

- number lines

- trios

- multiplication and division table


Make connections betw een arrays, number patterns and counting in steps of a constant size
Understand and use the inverse relationship betw een multiplication and division, including doubling and halving

## DIVISION

## Conceptual understanding and procedural fluency

To divide successfully, pupils need to:
recognise division as grouping or sharing
recall and use division facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
understand the link betw een division and fractions, and find fractions of a length, shape, set of objects or quantity calculate mathematical statements for division within the multiplication tables and write them using the division ( $\div$ ) and equals (=) signs

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:
solve problems involving division, using materials, arrays, repeated addition and subtraction, mental methods, and division facts, including problems in contexts

## Mental strategies

Use of models and images:

- concrete objects/pictorial representations

- arrays
$20 \div 5=4$
$20 \div 4=5$
- number lines

- trios
$4 \times 5=20$
$5 \times 4=20$
$20 \div 5=4$
$20 \div 4=5$

- multiplication and division table


Make connections betw een arrays, number patterns and counting in steps of a constant size
Understand and use the inverse relationship betw een multiplication and division, including doubling and halving

## Year 3

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:
read and write numbers up to 1000 in numerals and in words
count from 0 in multiples of $1,2,3,4,5,8,10,50$ and 100 , forw ards and backw ards
recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
identify, represent and estimate numbers using different representations
find 10 or 100 more or less than a given number
compare and order numbers up to 1000

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. $130+50=180$ continue to add numbers mentally, including:

- tw o tw o-digit numbers
- three or more one-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
add numbers with up to three digits, using the formal written method of column addition
estimate and check the answ er to a calculation, including using the inverse operation


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve problems, including missing number problems, using number facts, place value, and more complex addition

## Mental strategies

Use of models and images:

- $\quad$ Trios $7+5=12$ $5+7=12 \quad 12-$ 5=7 12-7=5

- the empty number line
- To multiples of 10 addition and subtraction tables

|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 |
| 20 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 30 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| 40 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 |
| 50 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 60 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| 70 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 |
| 80 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 |
| 90 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
| 100 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |


|  | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 10 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 |
| 20 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 |
| 30 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 |
| 40 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 |
| 50 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 |
| 60 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 |
| 70 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 |
| 80 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 |
| 90 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 |
| 100 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 |

Recognise and use the inverse relationship betw een addition and subtraction Use know ledge that addition can be done in any order (commutative), e.g.

- put the larger number first and count on in steps of 1, 10 or 100
- partition additions into hundreds, tens and ones, then recombine, e.g. $75+56=75+50+6$

$$
\begin{aligned}
& =125+6 \\
& =131
\end{aligned}
$$

Identify near doubles, using doubles already known, e.g. $70+71$
Add the nearest multiple of 10 or 100 , and adjust
Use patterns of similar calculations, e.g. $13+5=18$ and $130+50=180$

Use know ledge of the associative law when adding more than tw o numbers, e.g. $4+7+6=(4+6)+7$

$$
\begin{aligned}
& =10+7 \\
& =17
\end{aligned}
$$

## Written methods

Add numbers w ith up to three digits (HTO + HTO)
Estimate and check the answ er to a calculation

## Expanded written method

548+387

$$
\begin{array}{r}
548 \\
+387 \\
\hline 15 \\
120 \\
800 \\
\hline 935 \\
\hline
\end{array}
$$

The first stage in the written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. To find the partial sums either the ones or the hundreds can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method of columnar addition.

The addition of the tens in the calculation $548+387$ is described in the words 'forty add eighty equals one hundred and tw enty', stressing the link to the related fact 'four add eight equals tw elve'. The addition of the hundreds is described as 'five hundred add three hundred equals eight hundred', stressing the link to the related fact'five add three equals eight'.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## Formal written method of column addition

548+387

$$
\begin{array}{r}
548 \\
+387 \\
\hline 935 \\
\hline 11
\end{array}
$$

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.
Carry digits are recorded below the line, using the w ords 'carry ten' or 'carry one hundred', not ' carry one'.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
continue to recall and use subtraction facts to 20 fluently, and derive and use related facts, e.g. $120-70=50$
continue to subtract numbers mentally, including:

- two two-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
subtract numbers with up to three digits, using the formal written method of columnar subtraction
estimate and check the answ er to a calculation, including using the inverse operation


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
solve problems, including missing number problems, using number facts, place value, and more complex subtraction

## Mental strategies

Use of models and images:

- trios $7+5=$

- the empty number line

- Addition \& subtraction tables

|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 |
| 20 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 30 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| 40 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 |
| 50 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 60 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| 70 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 |
| 80 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 |
| 90 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
| 100 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |


|  | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 10 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 |
| 20 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 |
| 30 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 |
| 40 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 |
| 50 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 |
| 60 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 |
| 70 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 |
| 80 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 |
| 90 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 |
| 100 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 |

Recognise and use the inverse relationship betw een addition and subtraction
Find a difference by counting up from the smaller to the larger number
Subtract the nearest multiple of 10 or 100 , and adjust
Use patterns of similar calculations, e.g. $18-5=13$ and $180-50=130$
Use partitioning, e.g. $73-46=73-40-6$

$$
\begin{aligned}
& =33-6 \\
& =27
\end{aligned}
$$

## Written methods

Subtract numbers with up to three digits (HTO - HTO)
Estimate and check the answ er to a calculation

## Formal written method of columnar subtraction (decom position)

582-237

$$
\begin{array}{r}
72 \\
582 \\
-\quad 373 \\
\hline 45 \\
\hline
\end{array}
$$

Start by subtracting the least significant digits first, i.e. the ones, then the tens, and finally the hundreds. Refer to subtracting the tens, for example, by saying 'seventy subtract thirty', not 'seven subtract three'.

In this example the ones to be subtracted are larger than the ones you are subtracting from.
You exchange one of the 8 tens for 10 ones, crossing out 8 and writing a superscript 7 , and crossing out the 2 and $w$ riting a superscript 12 . The calculation then becomes 12 subtract 7,70 subtract 30 and finally 500 subtract 200 .
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## MULTIPLICATION

## Conceptual understanding and proce dural fluency

To multiply successfully, pupils need to:
consolidate recall of multiplication facts for the 2,5 and 10 multiplication tables
recall and use multiplication facts for the 3,4 and 8 multiplication tables
use known multiplication facts to derive related facts involving multiples of 10 , e.g. $2 \times 30=60$
$w$ rite and calculate mathematical statements for multiplication using the multiplication tables that they know, including for tw o-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods
estimate and check the answ er to a calculation, including using the inverse operation

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:
solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in whichn objects are connected to mobjects

## Mental strategies

Use of models and images:


- number lines

- trios $3 \times 4=12$
$4 \times 3=12$
$12 \div 4=3$
$12 \div 3=4$

- multiplication square


Make connections betw een arrays, number patterns and counting in steps of a constant size Understand and use the inverse relationship betw een multiplication and division
Use doubling, e.g. connect the 2, 4 and 8 multiplication tables

Use the 'key multiplication facts' of $\mathrm{x} 1, \times 2, \times 5$, and $\times 10$ to work out the answ ers to unknown multiplication facts, e.g. $7 \times 4=(5 \times 4)+(2 \times 4)$

$$
\begin{aligned}
& =20+8 \\
& =28
\end{aligned}
$$

Use patterns of similar calculations, e.g. $8 \times 6=48$ and $8 \times 60=480$
Show that multiplication of tw onumbers can be done in any order (commutative), e.g. $4 \times 12 \times 5=4 \times 5 \times 12$

$$
\begin{aligned}
& =20 \times 12 \\
& =240
\end{aligned}
$$

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit number by a one-digit number, $63 \times 8=(60 \times 8)+(3 \times 8)$

$$
\begin{aligned}
& =480+24 \\
& =504
\end{aligned}
$$

## Written methods

Short multiplication:

- Multiply a tw o-digit number by a one-digit number ( $\mathrm{TO} \times \mathrm{O}$ )

Estimate and check the answ er to a calculation
Expanded written method

| $63 \times 8$ |
| ---: |
| 63 |
| $\times \quad 8$ |
| 24 |
| 480 |
| $\frac{504}{1}$ |

The firststep is to show all of the calculations involved.
Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in $63 \times 8$ it is 'sixty multiplied by eight', not 'six multiplied by eight', although the relationship $6 \times 8$ should be stressed).
Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## Formal written method of short multiplication

| 63 |
| ---: |
| $\times \quad 28$ |

$\underline{504}$
The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## DIVISION

## Conceptual understanding and procedural fluency

To divide successfully, pupils need to:
consolidate recall of division facts for the 2,5 and 10 multiplication tables
recall and use division facts for the 3,4 and 8 multiplication tables
use know $n$ division facts to derive related facts involving multiples of 10, e.g. $60 \div 3=20$
w rite and calculate mathematical statements for division using the multiplication tables that they know develop reliable w ritten methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method of short division (without a remainder)
estimate and check the answ er to a calculation, including using the inverse operation

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to: solve problems, including missing number problems, involving division

## Mental strategies

Use of models and images:

- arrays

$12 \div 4=3$
$12 \div 3=4$

- trios $3 \times 4=12$
$4 \times 3=12$
$12 \div 4=3$
$12 \div 3=4$
- multiplication square


| $\times$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Make connections betw een arrays, number patterns and counting in steps of a constant size Understand and use the inverse relationship betw een multiplication and division
Use halving, e.g. find quarters by halving halves

Use patterns of similar calculations, e.g. $48 \div 8=6$ and $480 \div 80=6$
Understand and use the distributive law, e.g. partitioning when dividing a two-digit number by a one-digit number,

$$
92 \div 4=(80 \div 4)+(12 \div 4)
$$

$$
=20+3
$$

$$
=23
$$

## Written methods

Short division (w ithout a remainder):

- Divide a two-digit number by a one-digit number ( $\mathrm{TO} \div \mathrm{O}$ )

Estimate and check the answ er to a calculation

The firststep is to show all of the calculations involved.
Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answ er is a multiple of 10 ? (20) There are 20 fours or 80 , with 12 remaining. How many fours in 12 ? (3) So 92 divided by four is 23 .'

## Formal written method of short division

$$
\frac{23}{49^{1} 2}
$$

The expanded written method leads children to the formal written method of short division so that children fully
understand the procedure, and the effectiveness and efficiency of the method.
The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90 . It is $w$ ritten in front of the 2 to show that 12 now has to be divided by 4 .
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

## Year 4

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to: read and w rite numbers up to 10000 in numerals and in words
count in multiples of 1 to $10,25,50,100$ and 1000, forw ards or backw ards
recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
identify, represent and estimate numbers using different representations
find 10,100 or 1000 more or less than a given number
compare and order numbers beyond 1000
round any number to the nearest 10,100 or 1000

## DECIMALS

To add and subtract successfully, pupils need to:
recognise and write decimal equivalents of any number of tenths or hundredths
recognise the place value of each digit in a decimal to two decimal places
compare and order numbers w ith the same number of decimal places up to two decimal places
round decimals with one decimal place to the nearest $w$ hole number
understand the effect of multiplying and dividing a one-digit or two-digit number by 10 and 100

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
consolidate recall of addition facts to 20 and related facts involving multiples of 100 and 1000, e.g. $1300+500=1800$
and $500+1300=1800$
continue to add numbers mentally, including:

- two two-digit numbers
- three or more one-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
add numbers with up to four digits using the formal w ritten method of columnar addition where appropriate, including calculations involving money, e.g. £13.56 + £38.54
estimate and check the answ er to a calculation, including using the inverse operation


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve addition two-step problems in contexts, deciding which operations and methods to use and why solve simple measure and money problems involving decimals to two decimal places

## Mental strategies

Continue to use models and images when necessary:


- multiples of 10 addition and subtraction tables

|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 |
| 20 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 30 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| 40 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 |
| 50 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 60 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| 70 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 |
| 80 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 |
| 90 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
| 100 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |


|  | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 10 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 |
| 20 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 |
| 30 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 |
| 40 | 150 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 |
| 50 | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 |
| 60 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 |
| 70 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 |
| 80 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 |
| 90 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 |
| 100 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 |

Continue to use the relationship betw een addition and subtraction
Use know ledge of the commutative law, e.g.

- put the larger number first and count on in steps of 1,10 or 100
- partition additions into hundreds, tens and ones, then recombine, e.g. $356+57=356+50+7$

$$
\begin{aligned}
& =406+7 \\
& =413
\end{aligned}
$$

Identify near doubles, using doubles already known, e.g. $170+180$
Add the nearest multiple of 10,100 or 1000 , and adjust
Use patterns of similar calculations, e.g. $130+5=180$ and $1300+500=1800$
Use know ledge of the associative law when adding more than two numbers, e.g. $24+27+16=(24+16)+27$

$$
\begin{aligned}
& =40+27 \\
& =67
\end{aligned}
$$

## Written methods

Add numbers with up to four digits, including money and measures (ThHTO + ThHTO)
Estimate and check the answ er to a calculation

## Formal written method of colum nar addition

$2456+5378$
2456
$+\quad \underline{7834}$
11
Carry digits are recorded below the line, using the w ords 'carry ten', 'carry one hundred', or 'carry one thousand', not 'carry one'.
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.
If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal w ritten method of columnar addition.

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
consolidate recall of subtraction facts to 20 and related facts involving multiples of 100 and 1000 ,
e.g. $1800-500=1300$ and $1800-1300=500$
continue to subtract numbers mentally, including:

- two two-digit numbers
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate, including calculations involving money, e.g. £24.26-£17.58
estimate and check the answ er to a calculation, including using the inverse operation


## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to: solve subtraction tw o-step problems in contexts, deciding which operations and methods to use and why solve simple measure and money problems involving decimals to two decimal places

## Mental strategies

Continue to use models and images when necessary:


- multiples of 10 addition and subtraction tables


Continue to use the relationship betw een addition and subtraction
Calculate mentally a difference such as $5005-2998$ by counting up from the smaller to the larger number Subtract the nearest multiple of 10,100 or 1000 , and adjust
Use patterns of similar calculations, e.g. $18-5=13$ and $1800-500=1300$
Use partitioning, e.g. $456-84=456-80-4$

$$
\begin{aligned}
& =376-4 \\
& =372
\end{aligned}
$$

## Written methods

Subtract numbers with up to four digits, including money and measures (ThHTO - ThHTO)
Estimate and check the answ er to a calculation
Formal written method of columnar subtraction (decomposition)
6418-2546

```
51311
6 4 1 8
- 2546
```

Start by subtracting the least significant digits first, i.e. the ones, then the tens, then the hundreds and finally the thousands. Refer to subtracting the tens, for example, by saying ' 11 tens subtract four tens', not ' 11 subtract four'. In this example the tens and the hundreds to be subtracted are larger than both the tens and hundreds you are subtracting from.

The calculation begins 8 subtract 6 .
Then you exchange one of the 4 hundreds for 10 tens, crossing out 4 and writing a superscript 3 , and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 tens subtract 4 tens.
You then exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and $w$ riting a superscript 5, and writing a superscript 1 in front of the 3 to make 13 hundreds. The calculation then becomes 13 hundreds subtract 5 hundreds.

Then finally 5000 subtract 2000.
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

## MULTIPLICATION

## Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
consolidate recall of multiplication facts for the $2,3,4,5,8$ and 10 multiplication tables
recall and use multiplication facts for the $6,7,9,11$ and 12 multiplication tables
use know n multiplication facts to derive related facts involving multiples of 10 and 100, e.g. $200 \times 3=600$
use place value, know n and derived facts to multiply mentally, including: multiplying by 0 and 1 ; multiplying together three numbers
recognise and use factor pairs and commutativity in mental calculations
multiply two-digit and three-digit numbers by a one-digit number using formal written layout
estimate and check the answ er to a calculation, including using the inverse operation

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to: solve problems involving multiplying and adding, including using the distributive law and multiply two-digit numbers by one digit, including scaling problems and harder correspondence problems such as n objects are connected to m objects

Mental strategies

- Continue to use models and images:
- arrays
$6 \times 7=42$
$\star \star \star \star \star \star \star$
*     *         *             *                 *                     * 
*     *         *             *                 *                     *                         * 
* $\star \star \star \star \star \star$
$\star \star \star \star \star \star \star$
$\star \star \star \star \star \star \star$
- number lines
$7 \times 6=42$

- trios $7 \times 12=84$
$12 \times 7=84$
$84 \div 12=7$

$84 \div 7=12$
- multiplication square to $12 \times 12$

- Make connections betw een arrays, number patterns and counting in steps of a constant size
- Continue to use the inverse relationship betw een multiplication and division
- Continue to use doubling, e.g. connect the 3,6 and 12 multiplication tables
- Use the 'key multiplication facts' of $\times 1, \times 2, \times 5$, and $\times 10$ to work out the answ ers to unknow multiplication facts, e.g. $7 \times 9=(5 \times 9)+(2 \times 9)$

$$
=45+18
$$

$$
=63
$$

## Use closely related facts:

- multiply by 9 or 11 by multiplying by 10 and adjusting
- develop the $\times 12$ table by adding facts from the $\times 10$ and $\times 2$ table
- Use factors, e.g. $8 \times 14=8 \times 2 \times 7$ Use patterns of similar calculations, e.g. $8 \times 6=48$ and $8 \times 60=480$
- Understand and use the commutative law
- Understand and use the associative law, e.g. $6 \times 15=6 \times(5 \times 3)$

$$
\begin{aligned}
& =(6 \times 5) \times 3 \\
& =30 \times 3 \\
& =90
\end{aligned}
$$

- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, e.g.
$356 \times 7=(300 \times 7)+(50 \times 7)+(6 \times 7)$

$$
\begin{aligned}
& =2100+350+42 \\
& =2492
\end{aligned}
$$

## Written methods

Short multiplication:

- Multiply a two-digit or three-digit number by a one-digit number (TO $\times \mathrm{O} / \mathrm{HTO} \times \mathrm{O}$ ) Estimate and check the answ er to a calculation


## Expanded written method



The first step is to show all of the calculations involved.
Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in $356 \times 7$ it is 'fifty multiplied by seven', not 'five multiplied by seven', although the relationship $5 \times 7$ should be stres sed).
Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## Formal written method of short multiplication

$356 \times 7$

$$
\begin{array}{r}
356 \\
\times \quad 347 \\
\hline 2492 \\
\hline
\end{array}
$$

The expanded w ritten method leads to the formal w ritten method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## DIVISION

## Conceptual understanding and procedural fluency

To divide successfully, pupils need to:
consolidate recall of division facts for the $2,3,4,5,8$ and 10 multiplication tables
recall and use division facts for the $6,7,9,11$ and 12 multiplication tables
use know $n$ division facts to derive related facts involving multiples of 10 and 100, e.g. $600 \div 3=200$
use place value, know $n$ and derived facts to divide mentally, including dividing by 1
recognise and use factor pairs in mental calculations
divide tw o-digit and three-digit numbers by a one-digit number using formal written layout (without a remainder)
estimate and check the answ er to a calculation, including using the inverse operation

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:
solve problems involving division in contexts, deciding which operations and methods to use and why

## Mental strategies

Continue to use models and images:

```
- arrays
```




```
*********
```

*********
\star ** * * * *
\star ** * * * *
*
*
42\div6=7

```
42\div6=7
```

- number lines
- 



- trios

$$
\begin{aligned}
& 7 \times 12=84 \\
& 12 \times 7=84 \\
& 84 \div 12=7 \\
& 84 \div 7=12
\end{aligned}
$$



- multiplication square to $12 \times 12$


Make connections betw een arrays, number patterns and counting in steps of a constant size
Continue to use the inverse relationship betw een multiplication and division
Continue to use halving, e.g. connect the 3,6 and 12 multiplication tables

Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, $486 \div 9=(450 \div 9)+(36 \div 9)$

$$
\begin{aligned}
& =50+4 \\
& =54
\end{aligned}
$$

## Written methods

Short division (w ithout a remainder):

- Divide a two-digit or three-digit number by a one-digit number ( $\mathrm{TO} \div \mathrm{O} / \mathrm{HTO} \div \mathrm{O}$ )

Estimate and check the answ er to a calculation

## Expanded written method

$486 \div 9$

```
9\longdiv{486}
    450(50\times9)
    }
        6(4\times9)
```

The firststep is to show all of the calculations involved.
Children should describe what they are doing using phrases similar to the follow ing: 'How many nines divide into 480 so that the answ er is a multiple of 10 ? (50) There are 50 nines or 450 , w ith 36 remaining. How many nines in 36 ? (4) So 486 divided by nine is 54 .'

## Formal written method of short division

$\frac{5 \pi}{9148^{36}}$
The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 3 represents the 3 tens that are remaining after 9 has been divided into 480. It is written in front of the 6 to show that 36 now has to be divided by 9 .
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

## Year 5

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:
read, write, order and compare numbers to at least 1000000 and determine the value of each digit
count in multiples of 1 to $10,25,50,100$ and 1000, forw ards or backw ards
find $10,100,1000,10000$ or 100000 more or less than a given number
round any number up to 1000000 to the nearest $10,100,1000,10000$ and 100000

## DECIMALS

To add, subtract, multiply and divide successfully, pupils need to: read, write, order and compare numbers with up to three decimal places identify the value of each digit in numbers given to three decimal places
multiply and divide whole numbers and those involving decimals by 10,100 and 1000
recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
round decimals with tw o decimal places to the nearest whole number and to one decimal place

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
add numbers mentally with increasingly large numbers
add decimals, including a mix of one-digit whole numbers and tenths, e.g. $1.3+0.5$; and complements of 1 , e.g. $0.83+0.17$
add $w$ hole numbers $w$ ith more than four digits, including using the formal $w$ ritten method (columnar addition) add decimals to tw o places, including using the formal written method (columnar addition)
use rounding to estimate and check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve addition multi-step problems in contexts, deciding which operations and methods to use and why
solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
solve problems involving number up to three decimal places

## Mental strategies

Continue to use models and images when necessary:

- trios

- decimals addition and subtraction tables

|  | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| 0.1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 |
| 0.2 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 |
| 0.4 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 |
| 0.5 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
| 0.6 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| 0.7 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 |
| 0.8 | 0.8 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
| 0.9 | 0.9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 |
| 1 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2 |



Develop further the relationship betw een addition and subtraction
Continue to apply know ledge of the commutative law, e.g.

- put the larger number firstand count on in steps of $1,10,100$ or 1000 ;
- partition additions into hundreds, tens and ones, then recombine, e.g. $356+57=356+50+7$

$$
\begin{aligned}
& =406+7 \\
& =413
\end{aligned}
$$

Identify near doubles, using doubles already known, e.g. $1 \cdot 7+1 \cdot 8$
Add the nearest multiple of 10,100 or 1000 , and adjust
Use patterns of similar calculations, e.g. $9+7=16$ and $0.9+0 \cdot 7=1 \cdot 6$
Use know ledge of the associative law when adding more than tw o numbers, e.g. $24+27+16=(24+16)+27$

$$
\begin{aligned}
& =40+27 \\
& =67
\end{aligned}
$$

## Written methods

Add $w$ hole numbers $w$ ith more than four digits
Add decimals with up to tw o decimal places
Estimate and check the answ er to a calculation
Formal written method of columnar addition

$$
\begin{array}{rr}
12957+14635 \\
12957 \\
+14635 \\
\hline 27592 \\
\hline 11 & 56.47+84.84 \\
56.47 \\
\hline \quad 84.76 \\
\hline \frac{141 \cdot 33}{111}
\end{array}
$$

Carry digits are recorded below the line, using the w ords 'carry ten', 'carry one hundred', 'carry one thousand' or 'carry ten thousand', not 'carry one'.
Where appropriate, place value columns are labelled, e.g. TO-th, to remind children of the value of each of the digits.

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
subtract numbers mentally w ith increasingly large numbers
subtract decimals, including a mix of one-digit $w$ hole numbers and tenths, e.g. 1.8-0.7
subtract whole numbers with more than four digits, including using the formal written method (columnar subtraction) subtract decimals to tw o places, including using the formal written method (columnar subtraction) use rounding to estimate and check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
solve problems involving addition, subtraction, multiplication and division and a combination of these, including
understanding the meaning of the equals sign
solve problems involving numbers up to three decimal places

## Mental strategies

Continue to use models and images when necessary:

- trios
$7+5=12$
$5+7=12$
12-5=7
$12-7=5$

- decimals addition and subtraction tables


Develop further the relationship betw een addition and subtraction
Calculate mentally a difference such as $12462-2300$ by counting up from the smaller to the larger number Subtract the nearest multiple of 10,100 or 1000, and adjust
Use patterns of similar calculations, e.g. $16-9=7$ and $1.6-0.9=0.7$
Use partitioning, e.g. $456-84=456-80-4$

$$
\begin{aligned}
& =376-4 \\
& =372
\end{aligned}
$$

## Written methods

Subtract w hole numbers with more than four digits
Subtract decimals w ith up to two decimal places
Estimate and check the answ er to a calculation

## Formal written method of columnar subtraction (decomposition)

| $45257-17488$ | $83.72-36.49$ |
| :---: | :---: |
| 314111417 | $713 \quad 612$ |
| 45257 | 83.72 |
| $-\frac{17488}{27769}$ | $\frac{-36.49}{47.23}$ |

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying ' 14 tens subtract 8 tens', not ' 14 subtract 8 '.
In the first example the ones, tens, hundreds and thousands to be subtracted are all larger than all of the ones, tens, hundreds and thousands you are subtracting from.
The calculation begins by exchanging one of the 5 tens for 10 ones, crossing out the 5 and writing a superscript 4, and crossing out the 7 and writing a superscript 17. The calculation then becomes 17 subtract 8 .
You then exchange one of the 2 hundreds for 10 tens, crossing out the 2 and $w$ riting a superscript 1 , and $w$ riting a superscript 1 in front of the 4 to make 14 tens. The calculation then becomes 14 tens subtract 8 tens.
Next, you exchange one of the 5 thousands for 10 hundreds, crossing out the 5 and writing a superscript 4 , and writing a superscript 1 in front of the 1 to make 11 hundreds. The calculation then becomes 11 hundreds subtract 4 hundreds.
Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3 , and w riting a superscript 1 in front of the 4 to make 14 thousands. The calculation then becomes 14 thousands subtract 7 thousands.
Then finally 30000 subtract 10000 .
Where appropriate, place value columns are labelled, e.g. TO•th, to remind children of the value of each of the digits.

## MULTIPLICATION

## Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
consolidate recall of the multiplication facts for multiplication tables up to $12 \times 12$
use know $n$ multiplication facts to derive related facts involving multiples of 10,100 and 1000 , e.g. $70 \times 80=5600$
continue to use place value, know n and derived facts to multiply mentally
multiply $w$ hole numbers and those involving decimals by 10,100 and 1000
identify multiples and factors, including finding all factor pairs of a number, and common factors of tw onumbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19
recognise and use square numbers and cube numbers, and the notation for squared $\left({ }^{2}\right)$ and cubed $\left({ }^{3}\right)$ continue to recognise commutativity in mental calculations
multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for tw o-digit numbers
use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:
solve problems, involving multiplication including using their know ledge of factors and multiples, squares and cubes solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates

## Mental strategies

Continue to use models and images when necessary:

| arrays | ネ* $\star \star \star$ * * $\begin{gathered}\text { * } \\ \text { * } \\ \text { * }\end{gathered}$ * * $\star \star \star \star \star$ * $\star \star \star \star \star \star$ * $\star \star \star \star \star \star$ * * * * * * $\star$ |
| :---: | :---: |

- number line

- trios

- multiplication square to $12 \times 12$ /multiples of 10 multiplication square

| $*$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |


|  | 20 | 30 | 40 | 50 | 60 | 20 | 80 | 90 | 100 | 110 | 120 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 2 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| 3 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 |
| 4 | 80 | 120 | 160 | 200 | 240 | 280 | 320 | 360 | 400 | 440 | 480 |
| 5 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 |
| 6 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 | 600 | 660 | 720 |
| 7 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 | 700 | 770 | 840 |
| 8 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 | 800 | 880 | 960 |
| 9 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 | 900 | 990 | 1080 |
| 10 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 |
| 17 | 220 | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 |
| 72 | 240 | 360 | 480 | 600 | 720 | 840 | 960 | 1080 | 1200 | 1320 | 1440 |

Continue to use the inverse relationship betw een multiplication and division Use related facts and doubling and halving:

- double or halve the most significant digit first
- to multiply by 25 , multiply by 100 then divide by 4
- double one number and halve the other
- find the multiplication facts for the $\times 16$ multiplication table by doubling the $\times 8$ multiplication facts Use closely related facts:
- multiply by 19 or 21 by multiplying by 20 and adjusting
- develop the $\times 14$ multiplication table by adding facts from the $\times 12$ and $\times 2$ multiplication tables

Use factors, e.g. $9 \times 18=9 \times 6 \times 3$
Use patterns of similar calculations, e.g. $8 \times 6=48$ and $8 \times 600=4800$
Understand and use the commutative law, e.g. $14 \times 12=(2 \times 7) \times 12$

$$
\begin{aligned}
& =2 \times(7 \times 12) \\
& =2 \times 84 \\
& =168
\end{aligned}
$$

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one digit number, or tw o tw o-digit numbers, e.g.

$$
\begin{aligned}
& 378 \times 4=(300 \times 4)+(70 \times 4)+(8 \times 4) & 78 \times 34 & =(78 \times 30)+(78 \times 4) \\
= & 1200+280+32 & & =2340+312 \\
= & 1512 & & =2652
\end{aligned}
$$

## Short multiplication:

- Multiply numbers up to four digits by a one-digit number (HTO $\times \mathrm{O} / \mathrm{ThHTO} \times \mathrm{O}$ )

Estimate and check the answ er to a calculation

## Grid method

$378 \times 4$

|  | 300 | 70 | 8 |
| :--- | ---: | ---: | :--- |
| $\times$ | 1200 | 280 | $32=1512$ |

Expanded written method
$378 \times 4$

$$
378
$$


$32(8 \times 4)$
$280(70 \times 4)$
$1200(300 \times 4)$
1512

The firststep is to show all of the calculations involved.
Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in $378 \times 4$ it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship $7 \times 4$ should be stressed).
Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

## Formal written method of short multiplication

$378 \times 4$

$$
378
$$

$\begin{array}{r}\times 34 \\ \times \\ \hline\end{array}$
1512
The expanded w ritten method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Long multiplication:

- Multiply numbers up to three digits by a tw o-digit number (TO $\times$ TO/HTO $\times \mathrm{TO}$ )

Estimate and check the answ er to a calculation

## Grid method

$78 \times 44$

|  |  | 8 | 78 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 70 |  |  | 78 $\times \quad 3$ |  |
| 4 | 280 | 32 | $312 \rightarrow$ | 312 | $(78 \times 4)$ |
| 30 | 2100 | 240 | $+2340 \rightarrow$ | 2340 | $(78 \times 30)$ |
|  |  |  | $2652 \rightarrow$ | 2652 |  |

The firststep is to use the grid method to show all of the calculations involved and how this relates to the expanded w ritten method. For example, 78 is multiplied by 30 (using know ledge of $78 \times 3$ ), then 78 is multiplied by 4 , and finally the tw o products are added together.

## Expanded written method

$78 \times 34$
Multiplying the most significant digit first

78
$\times \quad 34$
$23240 \quad(78 \times 30)$
$(78 \times 4)$

Multiplying the least significant digit first

$$
\begin{aligned}
& 78 \\
& \times \quad 34 \\
& 3132(78 \times 4) \\
& 23240(78 \times 30)
\end{aligned}
$$

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

## DIVISION

Conceptual understanding and procedural fluency
To divide successfully, pupils need to:
consolidate recall of the division facts for multiplication tables up to $12 \times 12$
use know $n$ division facts to derive related facts involving multiples of 10,100 and 1000 , e.g. $6300 \div 90=70$ continue to use place value, know $n$ and derived facts to divide mentally divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answ ers up to three decimal places identify multiples and factors, including finding all factor pairs of a number, and common factors of tw onumbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 divide numbers up to 4 digits by a one-digit number using the formal w ritten method of short division and interpret remainders appropriately for the context
use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:
solve problems involving division, including using their know ledge of factors and multiples, squares and cubes
solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
solve problems involving division, including scaling by simple fractions and problems involving simple rates

## Mental strategies

Continue to use models and images when necessary:

- number lines

- trios
$7 \times 12=84$
$12 \times 7=84$
$84 \div 12=7$

$$
84 \div 7=12
$$



- multiplication square to $12 \times 12 /$ multiples of 10 multiplication square



## Continue to use the inverse relationship betw een multiplication and division

## Mental strategies continued

Continue to use halving, e.g. connect the 3,6 and 12 multiplication tables
Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, $486 \div 9=(450 \div 9)+(36 \div 9)$

$$
\begin{aligned}
& =50+4 \\
& =54
\end{aligned}
$$

## Written methods

Short division (including with remainders expressed as a w hole number, fraction or decimal):

- Divide numbers up to 4 digits by a one-digit number ( $\mathrm{HTO} \div \mathrm{O} / \mathrm{ThHTO} \div \mathrm{O}$ )

Estimate and check the answ er to a calculation

## Formal written method of short division

| Whole number remainder | Fraction remainder | Decimal remainder |
| :--- | :--- | :--- |
| $279 \div 6$ | $279 \div 6$ | $279 \div 6$ |
| $\frac{46 \mathrm{ra}}{6 \longdiv { 2 7 ^ { 3 9 } }}$ | $\frac{46 \frac{1}{2}}{27^{39}}$ | 46.5 |

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10 ? (40) There are 40 sixes or 240 , with 30 remaining.' The superscript 3 represents the 3 tens that are remaining after 6 has been divided into 270 . It is written in front of the 9 to show that 39 now has to be divided by 6 .
Children then ask: 'How many sixes in 39 ?' ( 6 remainder 3 ). Depending on the context, the remainder is written as a $w$ hole number, fraction, decimal or rounded up or dow $n$.

## Year 6

## NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:
read, w rite, order and compare numbers up to 10000000 and determine the value of each digit count in multiples of 1 to 10,25,50, 100 and 1000, forw ards or backw ards find $10,100,1000,10000$ or 100000 more or less than a given number round any whole number to a required degree of accuracy

## DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:
read, write, order and compare numbers with up to three decimal places
identify the value of each digit in numbers given to three decimal places
multiply and divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answ ers up to three decimal places
recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents round decimals with tw odecimal places to the nearest whole number and to one decimal place

## ADDITION

## Conceptual understanding and procedural fluency

To add successfully, pupils need to:
perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations practise addition for larger numbers and decimals, using the formal written method of columnar addition
use know ledge of the order of operations to carry out calculations involving the four operations
use estimation to check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
solve addition multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division
use estimation to check answ ers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
solve problems which require answ ers to be rounded to specified degrees of accuracy

## Mental strategies

Continue to use models and images when necessary:

- decimals addition and subtraction tables


Develop further the relationship betw een addition and subtraction
Continue to apply know ledge of the commutative law, e.g.

- put the larger number first and count on in steps of 1,10, 100 or 1000
- partition additions into hundreds, tens and ones, then recombine, e.g. $3356+257=3356+200+50+7$

$$
=3556+57
$$

Identify near doubles, using doubles already know n , e.g. $5 \cdot 7+5 \cdot 8$

$$
=3613
$$

Add the nearest multiple of 10,100 or 1000 , and adjust
Use patterns of similar calculations, e.g. $9+7=16$ and $0.09+0.07=0.16$
Use know ledge of the associative law when adding more than tw o numbers, e.g. $24+27+16=(24+16)+27$

$$
=40+27
$$

$$
=67
$$

## Written methods

Add numbers w ith more than four digits
Add decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
Estimate and check the answ er to a calculation

## Formal written method of columnar addition



Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand', ... not 'carry one'. Where appropriate, place value columns are labelled, e.g. TO. tht ${ }^{\text {h }}$, to remind children of the value of each of the digits.

## SUBTRACTION

## Conceptual understanding and procedural fluency

To subtract successfully, pupils need to: perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations practise subtraction for larger numbers and decimals, using the formal written method of columnar subtraction
use know ledge of the order of operations to carry out calculations involving the four operations
use estimation to check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division
use estimation to check answ ers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
solve problems which require answ ers to be rounded to specified degrees of accuracy

## Mental strategies

Continue to use models and images when necessary:

- decimals addition and subtraction tables


Develop further the relationship betw een addition and subtraction
Calculate mentally a difference such as $23004-18998$ by counting up from the smaller to the larger number
Subtract the nearest multiple of 10,100 or 1000 , and adjust
Use patterns of similar calculations, e.g. $16-9=7$ and $0.16-0.09=0.07$
Use partitioning, e.g. $4656-358=4656-300-50-8$

$$
\begin{aligned}
& =4356-58 \\
& =4298
\end{aligned}
$$

## Written methods

Subtract numbers with more than four digits
Subtract decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
Estimate and check the answ er to a calculation

Formal written method of columnar subtraction (decomposition)

| $746291-298354$ | $63.237-45.869$ |
| :---: | ---: |
| 6131512811 | 512111217 |
| 746291 | 63.237 |
| -298354 |  |
| 447937 | -45.869 |

Start by subtracting the least significant digits first, i.e. in the firstexample, the ones, then the tens ... and finally the hundreds of thousands. Refer to subtracting the tens, for example, by saying ' 8 tens subtract 5 tens', not ' 8 subtract 5 '.
In the first example, the ones, tens, thousands and tens of thousands to be subtracted are all larger than all of the ones, tens, thousands and tens of thousands you are subtracting from.
The calculation begins by exchanging one of the 9 tens for 10 ones, crossing out the 9 and $w$ riting a superscript 8 , and crossing out the 1 and $w$ riting a superscript 11 . The calculation then becomes 11 subtract 4 .
You then calculate 8 tens subtract 5 tens.
Next, you exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5 , and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 hundreds subtract 3 hundreds.
Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3 , and $w$ riting a superscript 1 in front of the 5 to make 15 thousands. The calculation then becomes 15 thousands subtract 8 thousands.
Next, you exchange one of the 7 hundreds of thousands for 10 tens of thousands, crossing out the 7 and writing a superscript 6 , and writing a superscript 1 in front of the 3 to make 13 tens of thousands. The calculation then becomes 13 t ens of thousands subtract 9 tens of thousands.
Then, finally, 600000 subtract 200000.
Where appropriate, place value columns are labelled, e.g. TO.tht ${ }^{h}$, to remind children of the value of each of the digits.

## MULTIPLICATION

## Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
consolidate recall of the multiplication facts for multiplication tables up to $12 \times 12$
use know $n$ multiplication facts to derive related facts involving multiples of 10,100 and 1000 , and decimals, e.g. $70 \times 80=5600,0.8 \times 6=4.8$
perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations continue to multiply whole numbers and those involving decimals by 10,100 and 1000
identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
establish whether a number up to 100 is prime and recall prime numbers up to 19
recognise and use square numbers and cube numbers, and the notation for squared $\left({ }^{2}\right)$ and cubed $\left({ }^{3}\right)$ continue to recognise commutativity in mental calculations
practise multiplication for larger numbers, using the formal written method of short multiplication multiply multi-digit numbers up to 4 digits by a tw o-digit whole number using the formal written method of long multiplication multiply one-digit numbers with up to tw o decimal places by whole numbers
multiply numbers with up to tw o decimal places by one- and two-digit whole numbers
use know ledge of the order of operations to carry out calculations involving the four operations
use estimation to check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:
solve problems involving addition, subtraction, multiplication and division
use estimation to check answ ers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
solve problems which require answ ers to be rounded to specified degrees of accuracy

## Mental strategies

Continue to use models and images when necessary:


- $\quad$ trios $7 \times 12=84$
$12 \times 7=84$
$84 \div 12=7$
$84 \div 7=12$

- multiplication square to $12 \times 12$ /multiples of 10 multiplication square/decimals multiplication square


|  | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 2 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| 3 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 |
| 4 | 80 | 120 | 160 | 200 | 240 | 280 | 320 | 360 | 400 | 440 | 480 |
| 5 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 |
| 6 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 | 600 | 660 | 720 |
| 7 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 | 700 | 770 | 840 |
| 8 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 | 800 | 880 | 960 |
| 9 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 | 900 | 990 | 1080 |
| 10 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 |
| 11 | 220 | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 |
| 12 | 240 | 360 | 480 | 600 | 720 | 840 | 960 | 1080 | 1200 | 1320 | 1440 |



Continue to use the inverse relationship betw een multiplication and division
Use related facts and doubling and halving:

- double or halve the most significant digit first
- to multiply by 25 , multiply by 100 then divide by 4
- double one number and halve the other
- find the multiplication facts for the $\times 24$ multiplication table by doubling the $\times 12$ multiplication facts
- squares of multiples of 10 to 100 , e.g. $70 \times 70$, and the corresponding halves
- doubles of decimals, e.g. $4.7 \times 2,0.63 \times 2$, and the corresponding halves
- doubles of multiples of 10 to 1000 , e.g. $830 \times 2$, and the corresponding halves
- doubles of multiples of 100 to 10000 , e.g. $48500 \times 2$, and the corresponding halves


## Use closely related facts:

- multiply by 49 or 51 by multiplying by 50 and adjusting
- develop the $\times 18$ multiplication table by adding facts from the $\times 10$ and $\times 8$ multiplication tables

Use factors, e.g. $9 \times 18=9 \times 6 \times 3$
Use patterns of similar calculations, e.g. $8 \times 6=48$ and $0.8 \times 6=4.8$
Continue to use and apply the commutative law
Understand and use the associative law, e.g. $10 \cdot 6 \times 30=10 \cdot 6 \times(10 \times 3)$ or

$$
=(10.6 \times 10) \times 3
$$

Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, or tw otwo-digit numbers, and partitioning when multiplying a whole number or decimal by a one-digit number, e.g.

$$
\begin{aligned}
285 \times 63 & =(200 \times 63)+(80 \times 63)+(5 \times 63) \\
& =12600+5040+315 \\
& =17955
\end{aligned}
$$

$$
\begin{aligned}
4 \cdot 83 & \times 6=(4 \times 6)+(0 \cdot 8 \times 6)+(0 \cdot 03 \times 6) \\
& =24+4 \cdot 8+0 \cdot 18 \\
& =28 \cdot 98
\end{aligned}
$$

## Written methods

Short multiplication (w hole numbers):

- Multiply multi-digit numbers up to 4 digits by a one-digit whole number

Estimate and check the answ er to a calculation

## Formal written method of short multiplication

$5643 \times 8$
5643
$\begin{array}{r}5328 \\ \hline 45144 \\ \hline\end{array}$
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the dig its.

Short multiplication (Decimals):

- Multiply one-digit or two-digit numbers with up to tw odecimal places by a one-digit number Estimate and check the answ er to a calculation


## Grid method

$4.83 \times 6$
$\begin{array}{lccc} & 4 & 0.8 & 0.03 \\ \times & 24 & 4.8 & 0.18\end{array}=28.98$

Expanded written method
$4.83 \times 6$ is equivalent to $483 \times 6 \div 100$


The first step is to show all of the calculations involved.
Children should describe what they do by referring to the actual values of the digits in the columns, e.g.

- $\quad$ when using the grid method and multiplying the tenths in $4.83 \times 6$ it is 'zero point eight multiplied by six', not 'eight multiplied by six'
- $\quad$ when using the expanded $w$ ritten method and multiplying the tens in $483 \times 6$ it is 'eighty multiplied by six, not 'eight multiplied by six'.
Although for both methods the relationship $8 \times 6$ should be stressed.
Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.


## Formal written method of short multiplication

$4.83 \times 6$ is equivalent to $483 \times 6 \div 100$
483
416
$\times \quad 4898$ 2898
$2898 \div 100=28.98$
The expanded w ritten method leads to the formal w ritten method of short multiplication so that children fully understand the
procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Long multiplication (w hole numbers):

- Multiply multi-digit numbers up to four digits by a two-digit number ( $\mathrm{TO} \times \mathrm{TO} / \mathrm{HTO} \times \mathrm{TO}$ )

Estimate and check the answ er to a calculation

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 285 is multiplied by 60 (using know ledge of $285 \times 6$ ), then 285 is multiplied by 3, and finally the two products are added together.

Expanded written method
Multiplying the most significant digit first


The grid method leads to the expanded w ritten method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the grid method_w ill depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.
Formal written method of long multiplication
$285 \times 63$
285
63
$\times \quad 8^{2} 5^{1} 5$
$17^{5} 1^{3} 00$
17955

The expanded w ritten method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.
Long multiplication (Decimals):

- Multiply one-digit numbers with up to two decimal places by a two-digit number

Estimate and check the answ er to a calculation

The firststep is to show all of the calculations involved, e.g. 7.56 is multiplied by 30 (using know ledge of $756 \times 3$ ), then 7.56 is multiplied by 4 , and finally the tw o products are added together.

## Method 2: Converting decimals to whole numbers before calculating, then con verting the answer back to decimals Expanded written method

Multiplying the most significant digit first
$7.56 \times 34$ is equivalent to $756 \times 34 \div 100$

## 756


$25704 \div 100=257.04$

| Multiplying the least significant digit first |  |  |
| :---: | :---: | :---: |
| $7.56 \times 34$ is equivalent to $756 \times 34 \div 100$ |  |  |
| 756 |  |  |
| 34 |  |  |
| $30^{2} 2^{2} 4$ | (756 | $\times 4$ |
| $22^{1} 6^{18} 80$ | (756 | $\times 30$ ) |
| 25704 |  |  |
| 1 |  |  |
| $25704 \div 100$ | 257 . |  |

$25704 \div 100=257 \cdot 04$

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## Formal written method of long multiplication

$7.56 \times 34$ is equivalent to $756 \times 34 \div 100$

$$
\begin{array}{r}
756 \\
\times \quad 34 \\
\hline 30^{2} 2^{2} 4 \\
22^{1} 6^{1} 80 \\
\hline 25704 \\
\hline 1
\end{array}
$$

$25704 \div 100=257.04$
The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

## DIVISION

To divide successfully, pupils need to:
consolidate recall of the division facts for multiplication tables up to $12 \times 12$
use know $n$ division facts to derive related facts involving multiples of 10,100 and 1000 , and decimals, e.g. $6300 \div 90=70$, $6 \cdot 3 \div 9=0 \cdot 7$
perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
continue to divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answ ers up to three decimal places
identify multiples and factors, including finding all factor pairs of a number, and common factors of tw onumbers
know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
establish whether a number up to 100 is prime and recall prime numbers up to 19
practise division for larger number, using the formal w ritten method of short division
divide numbers up to 4 digits by a tw o-digit w hole number using the formal w ritten method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
divide numbers up to 4 digits by a tw o-digit number using the formal w ritten method of short division where appropriate, interpreting remainders according to the context
divide numbers with up to tw o decimal places by one- and tw o-digit w hole numbers
use know ledge of the order of operations to carry out calculations involving the four operations
use estimation to check answ ers to calculations and determine, in the context of a problem, levels of accuracy

## Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:
solve problems involving addition, subtraction, multiplication and division
use estimation to check answ ers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
solve problems which require answ ers to be rounded to specified degrees of accuracy

## Mental strategies

Continue to use models and images when necessary:
number lines


- multiplication square to $12 \times 12 /$ multiples of 10 multiplication square/decimals multiplication square

| $\times$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |


| $\times$ | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 2 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| 3 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 |
| 4 | 80 | 120 | 160 | 200 | 240 | 280 | 320 | 360 | 400 | 440 | 480 |
| 5 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 |
| 6 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 | 600 | 660 | 720 |
| 7 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 | 700 | 770 | 840 |
| 8 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 | 800 | 880 | 960 |
| 9 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 | 900 | 990 | 1080 |
| 10 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 |
| 11 | 220 | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 |
| 12 | 240 | 360 | 480 | 600 | 720 | 840 | 960 | 1080 | 1200 | 1320 | 1440 |


|  | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 |
| 3 | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 | 2.1 | 2.4 | 2.7 | 3 | 3.3 | 3.6 |
| 4 | 0.8 | 1.2 | 1.6 | 2 | 2.4 | 2.8 | 3.2 | 3.6 | 4 | 4.4 | 4.8 |
| 5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 |
| 6 | 1.2 | 1.8 | 2.4 | 3 | 3.6 | 4.2 | 4.8 | 5.4 | 6 | 6.6 | 7.2 |
| 7 | 1.4 | 2.1 | 2.8 | 3.5 | 4.2 | 4.9 | 5.6 | 6.3 | 7 | 7.7 | 8.4 |
| 8 | 1.6 | 2.4 | 3.2 | 4 | 4.8 | 5.6 | 6.4 | 7.2 | 8 | 8.8 | 9.6 |
| 9 | 1.8 | 2.7 | 3.6 | 4.5 | 5.4 | 6.3 | 7.2 | 8.1 | 9 | 9.9 | 10.9 |
| 10 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 11 | 2.2 | 3.3 | 4.4 | 5.5 | 6.6 | 7.7 | 8.8 | 9.9 | 11 | 12.1 | 13.2 |
| 12 | 2.4 | 3.6 | 4.8 | 6 | 7.2 | 8.4 | 9.6 | 10.8 | 12 | 13.2 | 14.4 |

Continue to use the inverse relationship betw een multiplication and division
Continue to use halving, e.g. connect the 3,6 and 12 multiplication tables
Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, e.g. $486 \div 9=(450 \div 9)+(36 \div 9)$

$$
\begin{aligned}
& =50+4 \\
& =54
\end{aligned}
$$

## Written methods

Short division, including with remainders expressed as a whole number, fraction or decimal (whole numbers)

- Divide numbers up to 4 digits by a one-digit number ( $\mathrm{HTO} \div \mathrm{O} / \mathrm{ThHTO} \div \mathrm{O}$ )

Estimate and check the answ er to a calculation

## Formal written method of short division

Whole number remainder
$1838 \div 8$
229 r 6

Fraction remainder

$$
1838 \div 8
$$

$$
2 \quad 2 \quad 9 \frac{3}{4}
$$

8) $18 \mathrm{~B}^{2} \frac{7}{8}$

## Decimal remainder

$$
1838 \div 8
$$

$$
229.75
$$

$$
8 \longdiv { 1 1 8 7 8 }
$$

Children should describe what they are doing using phrases similar to the follow ing: 'How many eights divide into 1800 so that the answ er is a multiple of 100?' (200) There are 200 eights or 1600 , w ith 200 remaining. The superscript 2 represents the
2 hundreds that are remaining after 8 has been divided into 1800. It is written in front of the 3 to show that a total of 23 tens (230) now have to be divided by eight.

Children then ask: 'How many eights divide into 230 so that the answ er is a multiple of 10 ?' (20) There are 20 eights or 160 , with 70 remaining. The superscript 7 represents the 7 tens that are remaining after 8 has been divided into 230 . It is $w$ ritten in front of the 8 to show that 78 now has to be divided by 8 .
Children then ask: 'How many eights in 78 ?' ( 9 remainder 6). Depending on the context, the remainder is $w$ ritten as a whole number, fraction, decimal or rounded up or down.

## Short division (Decimals)

- Divide numbers with up to two decimal places by a one-digit number ( $\mathrm{O} \cdot \mathrm{th} \div \mathrm{O} / \mathrm{TO} \cdot \mathrm{th} \div \mathrm{O}$ )

Estimate and check the answ er to a calculation
Method 1: Calculating with decimals

$$
\frac{7 \cdot 5_{3} \varepsilon}{6 \longdiv { 4 5 . 3 6 }}
$$

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals $45 \cdot 36 \div 6$ is equivalent to $4536 \div 6 \div 100$

$756 \div 100=7.56$
Phrases similar to those above for short division of whole numbers should be used for short division involving decimals.
An emphasis should be placed on recognising the value of each of the digits in the dividend.

Long division, including with remainders expressed as a whole number, fraction or decimal (Whole numbers)

- Divide numbers up to 4 digits by a two-digit number (HTO $\div \mathrm{TO} / \mathrm{ThHTO} \div \mathrm{TO}$ )

Estimate and check the answ er to a calculation

Expanded written method of long division


Formal written method of long division

| $18)$$-\frac{54}{4} \\|_{3}^{36}$ |  |
| :---: | :---: |
|  |  |

```
5836 \div18=324 r 4 or %

The amount of time that should be spent teaching and practising the expanded written method of long division will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.

Long division (Decimals)
- Divide numbers with up to two decimal places by a two-digit whole number (TO.th \(\div \mathrm{TO}\) )

Estimate and check the answ er to a calculation

\section*{Method 1: Calculating with decimals}

Expanded written method of long division
\(58.32 \div 18\)


\section*{Formal written method of long division}
\(58.32 \div 18\)

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e Method 1) will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting whole and decimal numbers mentally, and in their understanding of place value.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method of long division
\(58.32 \div 18\) is equivalent to \(5832 \div 18 \div 100\)
\[
\begin{align*}
& \begin{array}{r}
324 \\
18 \lcm{5832}
\end{array} \\
& (300 \times 18) \\
& \frac{\left.5^{3}\right) 400}{432} \\
& -\frac{360}{7} \\
& -\quad 072
\end{align*}
\]

\section*{Formal written method of long division}
\(58.32 \div 18\) is equivalent to \(5832 \div 18 \div 100\)
\[
\begin{array}{r}
324 \\
1 8 \longdiv { 5 8 } \begin{array} { r } 
{ 3 2 } \\
{ - 5 4 } \\
{ 4 3 } \\
{ 3 }
\end{array} \\
\hline
\end{array}
\]```

